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EFFECTS OF ALIGNED MAGNETIC FIELD AND FIRST ORDER CHEMICAL REACTION ON AN UNSTEADY CONVECTIVE HEAT AND MASS TRANSFER FLOW PAST A VERTICAL PLATE EMBEDDED IN A POROUS MEDIUM

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ABSTRACT

This paper investigates the combined effects of aligned magnetic field and first order chemical reaction on unsteady convective heat and mass transfer flow past a vertical plate embedded in a porous medium is presented. At time $t' > 0$, the plate starts moving in its own plane with an impulsive velocity u_0 in the vertical upward direction against the gravitational field and at the same time, the plate temperature and concentration levels near the plate raised linearly with time t'. A transverse magnetic field of uniform strength B_0 is assumed to be applied at an angle α with the fluid flow direction. The resulting dimensionless coupled linear partial differential equations are solved by using Laplace transform technique. Effects of various flow parameters entering into the problem on flow quantities are discussed andanalyzed through graphs and tables.

Key Words: Chemical reaction,magnetohydrodynamics, mass transfer, thermal radiation, heat source, porous medium, vertical plate.

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NOMENCLATURE

A	A constant
a^*	Absorption coefficient
B_0	External magnetic field
C	Dimensionless concentration
C_P	Specific heat at constant pressure
C'	Species concentration
C'_w	Concentration near the plate
C'_∞	Concentration far away from the plate
D	Chemical molecular diffusivity
D_1	Coefficient of thermal diffusivity
G_m	Mass Grashof number
G_r	Thermal Grashof number
g	Acceleration due to gravity
H	Heat source parameter
T'_w	Temperature at the plate
T'_∞	Temperature of the fluid far away from the plate
t	Dimensionless time
t'	Time
u	Dimensionless velocity
u'	Velocity of the fluid in the x' - direction
u_0	Velocity of the plate
y	Dimensionless co-ordinateaxis normal to the plate
K	Dimensionless permeability parameter
K'	Permeability of the porous medium
K'_r	Chemical reaction parameter
M	Magnetic field parameter
Nu	Nusselt number
Pr	Prandtl number
Q'	Dimensional heat absorption coefficient
q_r	Radiative heat flux in the y' - direction
R	Radiative parameter
Sc	Schmidt number
S_O	Soret number
Sh	Sherwood number
T'	Temperature
y'	Co-ordinate axis normal to the plate

Greek symbols

α	Aligned angle
β	Volumetric coefficient of thermal expansion
β^*	Volumetric coefficient of concentration expansion
γ	Dimensionless chemical reaction parameter
κ	Thermal conductivity of the fluid

μ Coefficient of viscosity

υ Kinematic viscosity

ρ Density of the fluid

σ Electric conductivity

$\bar{\sigma}$ Stefan-Boltzmann constant

τ Shearing stress

θ Dimensionless fluid temperature

Subscripts

w Conditions on the wall

∞ Free stream conditions

1. INTRODUCTION

The study of the effect of magnetic field on free convective flows in electrolytes, liquid-metals and ionized gases is important. The influence of magnetic field on viscous, incompressible, electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads and sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads etc. in the presence of an electrically conducting fluid subject to a magnetic field.Manjulatha et al.[1] have studied the effect of an aligned magnetic field on two-dimensional MHD free convective flow of a viscous incompressible fluid through a porous medium confined in a vertical flat plate with temperature dependent heat absorption or generation. Aligned magnetic field and chemical reaction effects on flow past a vertical oscillating plate through porous medium are discussed by Sandeep and Sugunamma [2].

A large number of research works concerning transfer processes with chemical reactions have been reported. The chemical reaction effects depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it takes place at an interface and homogeneous if it take place in solution. In majority cases, a chemical reaction depends on the concentration of the species itself. In many chemical processes, a chemical reaction occurs between a foreign mass and a fluid in which a plate is moving. These processes take place in numerous industrial applications such as polymer production, manufacturing of ceramics or glassware and food processing.Chambre and Young [3] analyzed a first-order chemical reaction in the neighbourhood of a horizontal plate. Al-Azab and Al-Odat [4] investigatedthe influence ofthe chemical reaction on a transientMHD-free convection flow over a moving vertical porous plate.Mahapatra et al. [5] have studied effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface.

When the temperature of the surrounding fluid is rather high, radiation effects play an important role,and this situation exists in space technology applications such as cosmic flight aerodynamics, rocket propulsionsystems and plasma physics. In these cases, it is necessary to considerthe radiation effects in free convection flows.The flow through porousmedia occurs in the ground water hydrology, in irrigation and drainage problems, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs,in chemical engineering for filtration and purification process.The study of chemical reaction heat transfer in porous medium has important applications in tabular reactors, oxidation of solid materials, synthesis of ceramic materials etc.

Heat and mass transfer in a porous medium is prevalent in nature and many manmade technological processes. As a consequence the theory of flow through porous media has emerged as a vibrant discipline of intensive research activity. Recent developments in modern technology have intensified interest of many researchers in the study of simultaneous heat and mass transfer from different geometries through a porous medium in fluids. Muthucumaraswamy and Ganesan[6]investigated the effects of radiation on the flow past an impulsively started infinite vertical plate with variable temperatures using Laplace transform technique.The effects of chemical reaction and radiation absorption on free convective flow through a porous medium with a variable suction in the presence of uniform magnetic field were studied by SudheerBabu and Satyanarayana[7].Rajput and Surendra[8] discussed the MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion.The effects of mass transfer on flow past an impulsively started infinite vertical plate with Newtonian heating and chemical reaction was analyzed by Rajesh [9]. Narahari and Nayan [10] discussed free convection flow past an impulsively started infinite vertical plate with Newtonian heating in the presence of thermal radiation and mass diffusion.Kumar and Varma [11] examined the effects of thermal diffusion and radiation on unsteady MHD flow, through porous medium with variable temperature and mass diffusion in the presence of heat source/sink .

The Soret effect or thermophoresis is a phenomenon observed in mixtures of mobile particles where the different particle types exhibit different responses to the force of a temperature gradient. The term Soret effect most often applies to aerosol mixtures, but may also commonly refer to the phenomenon in all phases of matter. It has been used in commercial precipitators for applications similar to electro static precipitators, manufacturing of optical fibre in vapour deposition processes, facilitating drug discovery by allowing the detection of atamer binding by comparison of the bound verses unbound motion of the target molecule. It is also used to separate different polymers particles in fluid flow fractionation.Jha and Singh [12] presented an analytical study for free convection and mass transfer flow past an infinite vertical plate moving impulsively in its own plane taking soret effects into account.

In the present paper, the effects of aligned magnetic field, first order chemical reaction, thermal radiation and heat absorption on unsteady convective heat and mass transfer flow past a vertical plate embedded in a porous medium have been investigated.The governing dimensionless equations are solved by using the Laplace transform technique. The solutions are expressed in terms of exponential and complementary error functions.

2. MATHEMATICAL FORMULATION

An unsteady two-dimensional laminar free convective flow of a viscous, incompressible, electrically conducting, radiating fluid past an impulsively started infinite vertical plate with variable temperature and mass diffusion through porous medium in the presence of aligned magnetic field and first order chemical reaction are studied. The x'-axis is taken along the plate in the vertical upward direction and the y'-axis is taken normal to the plate. Initially it is assumed that the plate and fluid are at the sametemperature T'_∞ and concentration level C'_∞ in stationary condition for all the points.At time $t' > 0$, the plate is given animpulsive motion in the vertical upward direction against the gravitational field, such that it attains uniform velocity u_0 and at the same time, the plate temperature and concentration levels near the plate are raised linearly with time t'. A transverse magnetic field of uniform strength B_0 is assumed to be applied at an angle α with the fluid flow direction.The viscous dissipation and induced magnetic field are assumed to be negligible.The fluid considered here is gray, absorbing/emitting radiation but a non-scattering medium. Since the plate is assumed to be infinitely long in the x'-axis direction, all the physical variables are functions of y' and t' only. Applying the Boussinesq's approximation, the unsteady flow is governed by the following equations

$$\frac{\partial u'}{\partial t'}=g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)+\upsilon\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2\sin^2\alpha}{\rho}u'-\upsilon\frac{u'}{K'}$$

(1) $$\rho C_p\frac{\partial T'}{\partial t'}=\kappa\frac{\partial^2 T'}{\partial y'^2}-\frac{\partial q_r}{\partial y'}+Q'(T'_\infty-T')$$

(2) $$\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial y'^2}+D_1\frac{\partial^2 T'}{\partial y'^2}-K'_r(C'-C'_\infty)$$

(3)

with the following initial and boundary conditions

$$t'\le 0:\ u'=0,\ T'=T'_\infty,\ C'=C'_\infty,\quad \forall y'$$
$$t'>0:\ u'=u_0,\ T'=T'_\infty+(T'_w-T'_\infty)At',\ C'=C'_\infty+(C'_w-C'_\infty)At' \text{ at } y'=0$$
(4)
$$t'>0: u'\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty \text{ as } y'\to\infty$$

For an optically thin gray gas, the radiative heat flux q_r satisfies the following non-linear differential equation.

$$\frac{\partial q_r}{\partial y'}=-4a^*\sigma(T'^4_\infty-T'^4) \quad (5)$$

where a^* is the absorption coefficient and σ is the Stefan-Boltzmann constant. It is assumed that the temperature differences within the flow are sufficiently small such that

T'^4 may be expressed as a linear function of the fluid temperature T' using the Taylor's series about the free stream temperature T'_∞. After neglecting higher-order terms,

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty$$

(6)

Using Eqs.(5) and (6), Eq.(2) becomes

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + 16a^* \sigma T'^3_\infty (T'_\infty - T') + Q'(T'_\infty - T') \qquad (7)$$

We introduce thefollowingnon-dimensionalquantities

$$u = \frac{u'}{u_0},\ t = \frac{u_0^2 t'}{\upsilon},\ y = \frac{u_0 y'}{\upsilon}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Sc = \frac{\upsilon}{D},\ \Pr = \frac{\rho \upsilon C_p}{\kappa},$$

$$G_r = \frac{\upsilon g \beta (T'_w - T'_\infty)}{u_0^3}, G_m = \frac{\upsilon g \beta^* (C'_w - C'_\infty)}{u_0^3},\ M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, R = \frac{16a^* \upsilon^2 \sigma T'^3_\infty}{\kappa u_0^2},$$

(8) $$H = \frac{Q' \upsilon^2}{\kappa u_0^2},\ A = \frac{u_0^2}{\upsilon},\ K = \frac{K' u_0^2}{\upsilon},\ \gamma = \frac{\upsilon K'_r}{u_0^2},\ S_o = \frac{D_1 (T'_w - T'_\infty)}{\upsilon (C'_w - C'_\infty)}$$

In view of Eqs. (8), Eqs. (1), (7), and (3), respectively, reduce to the following non-dimensional form:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m C - M_1 u$$

(9) $$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{1}{\Pr} (R + H) \theta$$

(10) $$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + S_o \frac{\partial^2 \theta}{\partial y^2} - \gamma C$$

(11)

with the following initial and boundary conditions

$$t \le 0 : u = 0,\ \theta = 0,\ C = 0\ \ \forall y$$

$t > 0: u = 1,\ \theta = t\ ,\ \ C = t$ at $y = 0$

(12)

$t > 0: u \to 0\ ,\ \theta \to 0\ ,\ C \to 0$ as $y \to \infty$

3. SOLUTION OF THE PROBLEM

The physical parameters that appear are defined in the nomenclature. The governing dimensionless equations from (9) to (11), subject to the initial and boundary conditions (12) are solved by the usual Laplace transform technique and the solutions for velocity, temperature and concentration fields are expressed in terms of exponential and complementary error functions which are given below:.

$$u(y,t) = \frac{1}{2}\left[\exp\left(y\sqrt{M_1}\right) erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{M_1 t}\right) + \exp\left(-y\sqrt{M_1}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{M_1 t}\right)\right]$$

$$+ A_1\left[\left(\frac{t}{2} + \frac{y\Pr}{4\sqrt{S}}\right)\exp\left(y\sqrt{S}\right) erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} + \sqrt{\frac{St}{\Pr}}\right) + \left(\frac{t}{2} - \frac{y\Pr}{4\sqrt{S}}\right)\exp\left(-y\sqrt{S}\right) erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} - \sqrt{\frac{St}{\Pr}}\right)\right]$$

$$+ A_2\left[\left(\frac{t}{2} + \frac{y\sqrt{Sc}}{4\sqrt{\gamma}}\right)\exp\left(y\sqrt{\gamma\, Sc}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{\gamma t}\right) + \left(\frac{t}{2} - \frac{y\sqrt{Sc}}{4\sqrt{\gamma}}\right)\exp\left(-y\sqrt{\gamma\, Sc}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{\gamma t}\right)\right]$$

$$-(A_1 + A_2)\left[\left(\frac{t}{2} + \frac{y}{4\sqrt{M_1}}\right)\exp\left(y\sqrt{M_1}\right) erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{M_1 t}\right) + \left(\frac{t}{2} - \frac{y}{4\sqrt{M_1}}\right)\exp\left(-y\sqrt{M_1}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{M_1 t}\right)\right]$$

$$+ \frac{A_3}{2}\exp(-dt)\left[\exp\left(y\sqrt{S - d\Pr}\right) erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} + \sqrt{\left(\frac{S}{\Pr} - d\right)t}\right) + \exp\left(-y\sqrt{S - d\Pr}\right) erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} - \sqrt{\left(\frac{S}{\Pr} - d\right)t}\right)\right]$$

$$+ \frac{A_4}{2}\exp(-dt)\left[\exp\left(y\sqrt{(\gamma - d)\, Sc}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(\gamma - d)t}\right) + \exp\left(-y\sqrt{(\gamma - d)\, Sc}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(\gamma - d)t}\right)\right]$$

$$+ \frac{A_5}{2}\exp(-lt)\left[\exp\left(y\sqrt{M_1 - l}\right) erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(M_1 - l)t}\right) + \exp\left(-y\sqrt{M_1 - l}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(M_1 - l)t}\right)\right]$$

$$- \frac{A_5}{2}\exp(-lt)\left[\exp\left(y\sqrt{S - l\Pr}\right) erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} + \sqrt{\left(\frac{S}{\Pr} - l\right)t}\right) + \exp\left(-y\sqrt{S - l\Pr}\right) erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} - \sqrt{\left(\frac{S}{\Pr} - l\right)t}\right)\right]$$

$$+\frac{A_6}{2}\exp(-nt)\left[\exp\left(y\sqrt{M_1-n}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M_1-n)t}\right)+\exp\left(-y\sqrt{M_1-n}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M_1-n)t}\right)\right]$$

$$-\frac{A_6}{2}\exp(-nt)\left[\exp\left(y\sqrt{(\gamma-n)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(\gamma-n)t}\right)+\exp\left(-y\sqrt{(\gamma-n)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(\gamma-n)t}\right)\right]$$

$$+\frac{A_7}{2}\left[\exp\left(y\sqrt{S}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{St}{\Pr}}\right)+\exp\left(-y\sqrt{S}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{St}{\Pr}}\right)\right]$$

$$+\frac{A_8}{2}\left[\exp\left(y\sqrt{\gamma Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{\gamma t}\right)+\exp\left(-y\sqrt{\gamma Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{\gamma t}\right)\right]$$

$$-\frac{1}{2}(A_7+A_8)\left[\exp\left(y\sqrt{M_1}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{M_1t}\right)+\exp\left(-y\sqrt{M_1}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{M_1t}\right)\right]$$

$$\theta(y,t)=\left(\frac{t}{2}+\frac{y\Pr}{4\sqrt{S}}\right)\exp\left(y\sqrt{S}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{St}{\Pr}}\right)+\left(\frac{t}{2}-\frac{y\Pr}{4\sqrt{S}}\right)\exp\left(-y\sqrt{S}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{St}{\Pr}}\right)$$

$$C(y,t)=(1+f)\left[\left(\frac{t}{2}+\frac{y\sqrt{Sc}}{4\sqrt{\gamma}}\right)\exp\left(y\sqrt{\gamma Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{\gamma t}\right)+\left(\frac{t}{2}-\frac{y\sqrt{Sc}}{4\sqrt{\gamma}}\right)\exp\left(-y\sqrt{\gamma Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{\gamma t}\right)\right]$$

$$+\frac{1}{2}\left(g-\frac{f}{d}\right)\left[\exp\left(y\sqrt{\gamma Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{\gamma t}\right)+\exp\left(-y\sqrt{\gamma Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{\gamma t}\right)\right]$$

$$-\frac{1}{2}\left(g-\frac{f}{d}\right)\exp(-dt)\left[\exp\left(y\sqrt{(\gamma-d)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(\gamma-d)t}\right)+\exp\left(-y\sqrt{(\gamma-d)Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(\gamma-d)t}\right)\right]$$

$$-\frac{1}{2}\left(g-\frac{f}{d}\right)\left[\exp\left(y\sqrt{S}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{St}{\Pr}}\right)+\exp\left(-y\sqrt{S}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{St}{\Pr}}\right)\right]$$

$$-f\left[\left(\frac{t}{2}+\frac{y\Pr}{4\sqrt{S}}\right)\exp\left(y\sqrt{S}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{St}{\Pr}}\right)+\left(\frac{t}{2}-\frac{y\Pr}{4\sqrt{S}}\right)\exp\left(-y\sqrt{S}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{St}{\Pr}}\right)\right]$$

$$+\frac{1}{2}\left(g-\frac{f}{d}\right)\exp(-dt)\left[\exp\left(y\sqrt{S-d\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\left(\frac{S}{\Pr}-d\right)t}\right)+\exp\left(-y\sqrt{S-d\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\left(\frac{S}{\Pr}-d\right)t}\right)\right]$$

where

$$M_1 = M\sin^2\alpha + \frac{1}{K},\ S = R + H,\ b = S_o Sc,\ c = \frac{S}{\Pr - Sc},\ d = \frac{S - \gamma Sc}{\Pr - Sc},\quad f = \frac{bc}{d},\ g = \frac{f\Pr}{S},$$

$$l = \frac{S - M_1}{\Pr - 1},\ n = \frac{\gamma Sc - M_1}{Sc - 1},\quad A_1 = \frac{bcG_m - dG_r}{d(S - M_1)},\ A_2 = -\frac{(d + bc)G_m}{d(\gamma Sc - M_1)},$$

$$A_3 = \frac{bcG_m(S - d\Pr)}{d^2 S(S - M_1 + d - d\Pr)},\ A_4 = -\frac{bcG_m(S - d\Pr)}{d^2 S(\gamma Sc - M_1 + d - dSc)},$$

$$A_5 = \frac{(\Pr - 1)[G_r S(S - M_1 + d - d\Pr) + bcG_m(M_1\Pr - S)]}{S(S - M_1)^2(S - M_1 + d - d\Pr)},$$

$$A_6 = \frac{(Sc - 1)G_m[(\gamma Sc - M_1)(S + bc\Pr) - (d + bc)S(Sc - 1)]}{S(\gamma Sc - M_1)^2(\gamma Sc - M_1 + d - dSc)},$$

$$A_7 = \frac{dS(\Pr - 1)(dG_r - bcG_m) + bcG_m(S - M_1)(d\Pr - S)}{d^2 S(S - M_1)^2},$$

$$A_8 = \frac{G_m[d^2 S(Sc - 1) + bcd\Pr M_1 - bcS(M_1 + d - dSc) + bc\gamma Sc(S - d\Pr)]}{d^2 S(\gamma Sc - M_1)^2}$$

SKIN FRICTION:

The boundary layer produces a drag force on the plate due to the viscous stresses which are developed at the wall. The viscous stress at the surface of the plate is given by

$$\tau = -\left[\frac{\partial u}{\partial y}\right]_{y=0}$$

$$\tau = \frac{1}{\sqrt{\pi t}}\exp(-M_1 t) + \sqrt{M_1}\, erf\left(\sqrt{M_1 t}\right) + A_1\left[\sqrt{\frac{t\Pr}{\pi}}\exp\left(-\frac{St}{\Pr}\right) + \left(t\sqrt{S} + \frac{\Pr}{2\sqrt{S}}\right) erf\left(\sqrt{\frac{St}{\Pr}}\right)\right]$$

$$+ A_2\left[\sqrt{\frac{tSc}{\pi}}\exp(-\gamma t) + \sqrt{Sc}\left(t\sqrt{\gamma} + \frac{1}{2\sqrt{\gamma}}\right) erf\left(\sqrt{\gamma t}\right)\right]$$

$$-(A_1 + A_2)\left[\sqrt{\frac{t}{\pi}}\exp(-M_1 t) + \left(t\sqrt{M_1} + \frac{1}{2\sqrt{M_1}}\right) erf\left(\sqrt{M_1 t}\right)\right]$$

$$+ A_3\exp(-dt)\left[\sqrt{\frac{\Pr}{\pi t}}\exp\left(-\left(\frac{S}{\Pr} - d\right)t\right) + \sqrt{S - d\Pr}\ erf\left(\sqrt{\left(\frac{S}{\Pr} - d\right)t}\right)\right]$$

$$+A_4\exp(-dt)\left[\sqrt{\frac{Sc}{\pi t}}\exp\left(-(\gamma-d)t\right)+\sqrt{(\gamma-d)Sc}\ erf\left(\sqrt{(\gamma-d)t}\right)\right]$$

$$+A_5\exp(-lt)\left[\frac{1}{\sqrt{\pi t}}\exp(-(M_1-l)t)+\sqrt{M_1-l}\ erf\left(\sqrt{(M_1-l)t}\right)\right]$$

$$-A_5\exp(-lt)\left[\sqrt{\frac{\Pr}{\pi t}}\exp\left(-\left(\frac{S}{\Pr}-l\right)t\right)+\sqrt{S-l\Pr}\ erf\left(\sqrt{\left(\frac{S}{\Pr}-l\right)t}\right)\right]$$

$$+A_6\exp(-nt)\left[\frac{1}{\sqrt{\pi t}}\exp(-(M_1-n)t)+\sqrt{M_1-n}\ erf\left(\sqrt{(M_1-n)t}\right)\right]$$

$$-A_6\exp(-nt)\left[\sqrt{\frac{Sc}{\pi t}}\exp\left(-(\gamma-n)t\right)+\sqrt{(\gamma-n)Sc}\ erf\left(\sqrt{(\gamma-n)t}\right)\right]$$

$$+A_7\left[\sqrt{\frac{\Pr}{\pi t}}\exp\left(-\frac{St}{\Pr}\right)+\sqrt{S}\ erf\left(\sqrt{\frac{St}{\Pr}}\right)\right]$$

$$+A_8\left[\sqrt{\frac{Sc}{\pi t}}\exp\left(-\gamma t\right)+\sqrt{\gamma Sc}\ erf\left(\sqrt{\gamma t}\right)\right]$$

$$-(A_7+A_8)\left[\frac{1}{\sqrt{\pi t}}\exp(-M_1 t)+\sqrt{M_1}\ erf\left(\sqrt{M_1 t}\right)\right]$$

NUSSELT NUMBER:

From temperature field, now we study Nusselt number (rate of change of heat transfer) which is given in non-dimensional form as

$$Nu=-\left[\frac{\partial\theta}{\partial y}\right]_{y=0}$$

$$Nu=\sqrt{\frac{t\Pr}{\pi}}\exp\left(-\frac{St}{\Pr}\right)+\left(t\sqrt{S}+\frac{\Pr}{2\sqrt{S}}\right)erf\left(\sqrt{\frac{St}{\Pr}}\right)$$

SHERWOOD NUMBER:

From concentration field, now we study Sherwood number (rate of change of mass transfer) which is given in non-dimensional form as

$$Sh = -\left[\frac{\partial C}{\partial y}\right]_{y=0}$$

$$Sh = (1+f)\left[\sqrt{\frac{tSc}{\pi}}\exp(-\gamma t) + \sqrt{Sc}\left(t\sqrt{\gamma} + \frac{1}{2\sqrt{\gamma}}\right)erf\left(\sqrt{\gamma t}\right)\right]$$

$$+\left(g-\frac{f}{d}\right)\left[\sqrt{\frac{Sc}{\pi t}}\exp(-\gamma t) + \sqrt{\gamma\, Sc}\; erf\left(\sqrt{\gamma t}\right)\right]$$

$$-\left(g-\frac{f}{d}\right)\exp(-dt)\left[\sqrt{\frac{Sc}{\pi t}}\exp(-(\gamma-d)t) + \sqrt{(\gamma-d)Sc}\; erf\left(\sqrt{(\gamma-d)t}\right)\right]$$

$$-\left(g-\frac{f}{d}\right)\left[\sqrt{\frac{\Pr}{\pi t}}\exp\left(-\frac{St}{\Pr}\right) + \sqrt{S}\; erf\left(\sqrt{\frac{St}{\Pr}}\right)\right]$$

$$-f\left[\sqrt{\frac{t\Pr}{\pi}}\exp\left(-\frac{St}{\Pr}\right) + \left(t\sqrt{S} + \frac{\Pr}{2\sqrt{S}}\right)erf\left(\sqrt{\frac{St}{\Pr}}\right)\right]$$

$$+\left(g-\frac{f}{d}\right)\exp(-dt)\left[\sqrt{\frac{\Pr}{\pi t}}\exp\left(-\left(\frac{S}{\Pr}-d\right)t\right) + \sqrt{S-d\Pr}\; erf\left(\sqrt{\left(\frac{S}{\Pr}-d\right)t}\right)\right]$$

4. RESULTS AND DISCUSSION

In order to get a physical insight into the problem, numerical computations are carried out to illustrate the effects of different governing parameters upon the nature of the flow and transport. The numerical values of the velocity, temperature, concentration,skin-friction, Nusselt number and Sherwood number are computed for different physical parameters e.g.magnetic field parameter (M) ,thermal Grashof number (G_r) , mass Grashof number (G_m) ,permeability parameter (K) , radiation parameter (R) , heat source parameter (H) , Prandtl number $(\Pr)$, Schmidt number (Sc) ,Soret number (S_o) ,chemical reaction rarameter (γ) ,time (t) and aligned angle (α). A representative set of numerical results is shown graphically in figures 1-15 and tables 1-5 for the cases of heating $(G_r < 0, G_m < 0)$ and cooling $(G_r > 0, G_m > 0)$ of the plate at time $t = 0.2$ or $t = 0.4$. Heating and cooling take place by virtueof the free convection current due to the temperature and concentration gradients.The default values for the control parameters are taken as: $\Pr = 0.71$ (air), 7.0 (water), $Sc = 0.78$ (ammonia), 0.96 (carbondioxide), 2.01 (ethyl benzene), aligned angle $\alpha = \frac{\pi}{3}$ and the other parameters are arbitrarily chosen. The present study with $\alpha = \frac{\pi}{2}$ and small values of chemical reaction parameter γ gives a good agreement with the numerical values of Kumar and Varma [11]and is shown in table-5.

Fig. 1 presents the effect of magnetic field parameter on fluid velocityin the cases of cooling and heating of the plate for $\Pr = 0.71$. As expected, we observed that for an increase in magneticfield parameter the velocity decreases in both the cases.Fig. 2reveals the velocity profiles for different values of Soret number in both cases of cooling and heating of the plate. Itis seen that the fluid velocity increases with increasing values of S_o in case of cooling of the plate and a reverse effectis observed in the case of heating of the plate.

Figs. 3 and 4 show the effects ofthermal Grashof number (G_r), mass Grashof number (G_m) and time (t) on the velocity field u. From these figures it is seen that the velocity u increasesas G_r or G_m or t increases in case of cooling of the plate, and an opposite tendency takes place in the case of heating.From Fig. 5it is seen thatin both cases of cooling and heating, the velocity u increases as permeability parameter K increases. From tables 1-4 it is observed that with the increase ofradiation parameter or heat source parameter the velocity increases up to certain y value (distance from the plate)and decreases later for the case of cooling of the plate. But the trend is just reversed in case of heating of the plate.

Fig. 6 displays the velocity profiles for different values of chemical reaction parameter (γ) in the cases of cooling and heating of the plate. As expected, the presence of a chemical reaction significantly affects the velocity profiles. It should be mentioned that the studied case refers to a destructive $(\gamma > 0)$ chemical reaction. In fact, as γ increases, a considerable reduction in the velocity is predicted for $\Pr = 0.71$ in the case of cooling of the plate. However, a reverse effect is observed in the case of heating of the plate. The effect of aligned angle (α) on the velocity field in cases of cooling and heating of the plate is shown in Fig.7. It is seen that the velocity u decreases with increasing α in both cases of cooling and heating of the plate.

The effects of Radiation parameter (R), Heat source parameter (H) on temperature of the flow field are shown in Fig. 8. It is noticed that as R or H increases the temperature of the flow fielddecreases at all the points in flow region.Concentration profiles of the flow field are displayed through Figs.9, 10, 11 and 12. From Fig. 9it is observed that the concentration increases with an increase in S_o. Figs. 10 and 11 reveal the effectsof Sc and R on the concentration distribution of the flow field. The concentration distribution is found to increasefaster up to certain y value (distance from the plate) and decreases later as Sc or R become heavier. From Fig. 12 it is noticed thatthe concentration decreases with increasing valuesof γ.

The skin-friction against time t for different values of the parameters is presented in Fig. 13. It is seen that the skin-friction increases as M increases and decreases with increase in S_o or Sc or K for $\Pr = 0.71$in the case of cooling of the plate, and the effect is reversed in the case of heating of the plate. Fig. 14illustrates the variation of Nusselt number against time t. It is noticed that the Nusselt number increases in the presence of radiation or as $\Pr$ increases. Also we observed that the Nusselt number for water $(\Pr = 7.0)$ ishigher than that of air $(\Pr = 0.71)$. Finally,it is found, from Fig. 15, that the Sherwood number decreases with increase in Schmidt number,Soret number and radiation parameter.

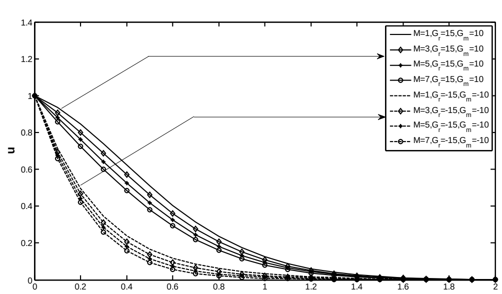

Fig. 1: Velocity profiles for different M with $S_o = 5, Sc = 2.01, \Pr = 0.71, K = 0.5, R = 10, H = 4, \gamma = 1, \alpha = \frac{\pi}{3}, t = 0.2$

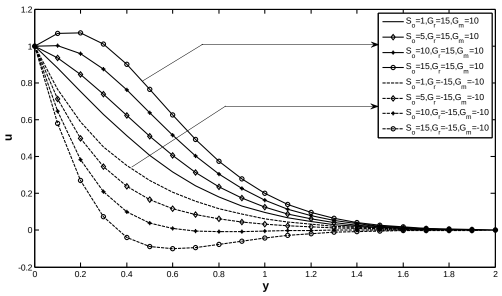

Fig. 2: Velocity profiles for different S_o with $M=3, Sc=2.01, \Pr=0.71, K=0.5,$ $R=10, H=4, \gamma=1, \alpha=\frac{\pi}{3}, t=0.2$

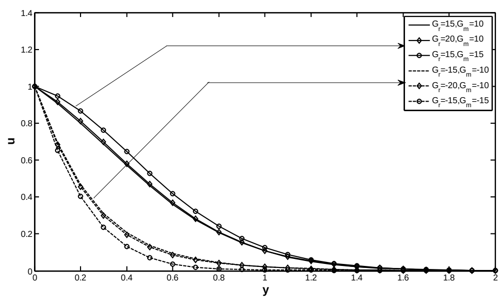

Fig. 3: Velocity profiles for different G_r & G_m with $M=3, S_o=5, Sc=2.01, \Pr=0.71,$ $K=0.5, R=10, H=4, \gamma=1, \alpha=\frac{\pi}{3}, t=0.2$

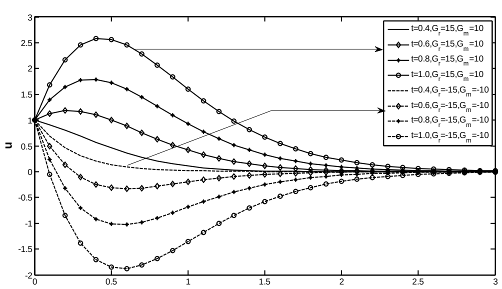

Fig. 4: Velocity profiles for different t with $M=3, S_o=5, Sc=2.01, \Pr=0.71,$ $K=0.5, R=10, H=4, \gamma=1, \alpha=\frac{\pi}{3}$

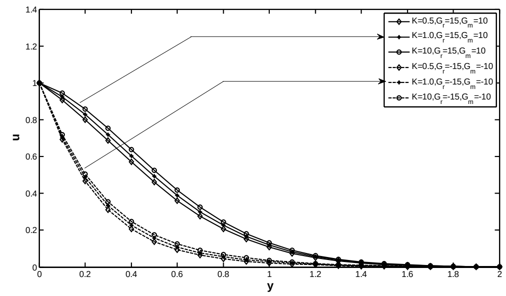

Fig. 5: Velocity profiles for different K with $M=3, S_o=5, Sc=2.01, \Pr=0.71,$ $R=10, H=4, \gamma=1, \alpha=\frac{\pi}{3}, t=0.2$

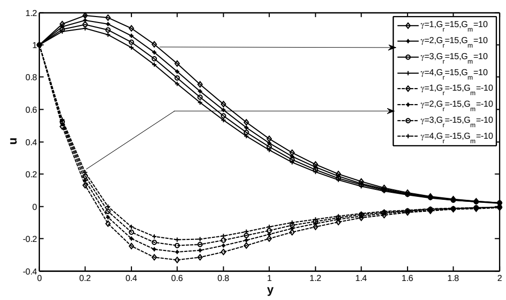

Fig. 6: Velocity profiles for different γ with $M=3, S_o=5, Sc=2.01, \Pr=0.71,$ $K=0.5, R=10, H=4, \alpha=\frac{\pi}{3}, t=0.2$

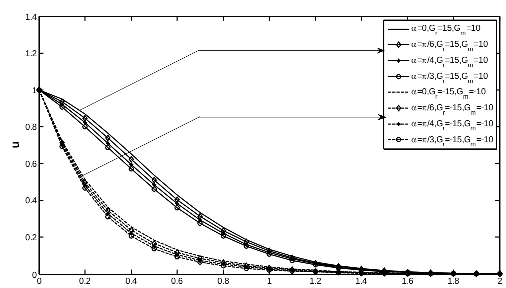

Fig. 7: Velocity profiles for different α with $M=3, S_o=5, Sc=2.01, \Pr=0.71,$ $K=0.5, R=10, H=4, \gamma=1, t=0.2$

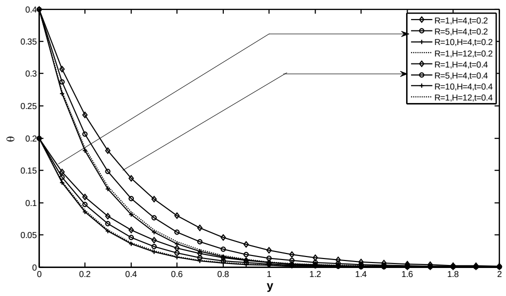

Fig. 8: Temperature profiles for different R and H

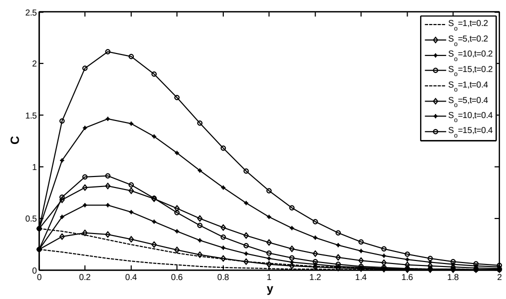

Fig. 9: Concentration profiles for different S_o with $R = 4, H = 1,$ $Sc = 2.01, \Pr = 0.71, \gamma = 1$

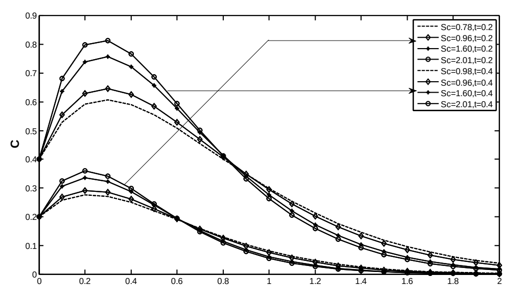

Fig. 10: Concentration profiles for different Sc with $R = 4, H = 1,$ $S_o = 5, \Pr = 0.71, \gamma = 1$

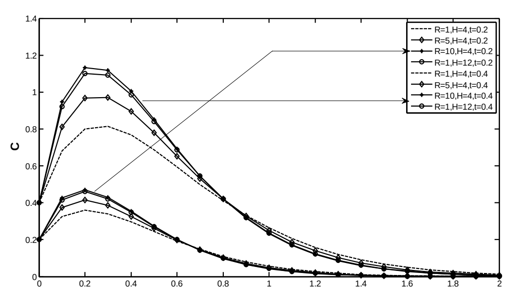

Fig. 11: Concentration profiles for different R and H with $Sc = 2.01, S_o = 5, \Pr = 0.71, \gamma = 1$

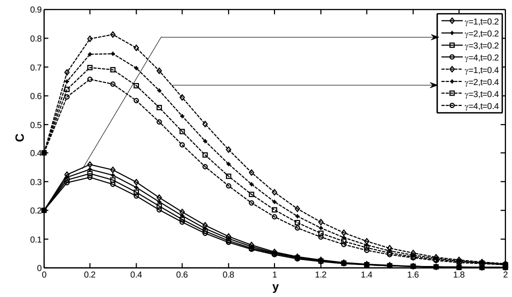

Fig. 12: Concentration profiles for different γ with $R = 4, H = 1,$ $S_o = 5, \Pr = 0.71, Sc = 2.01$

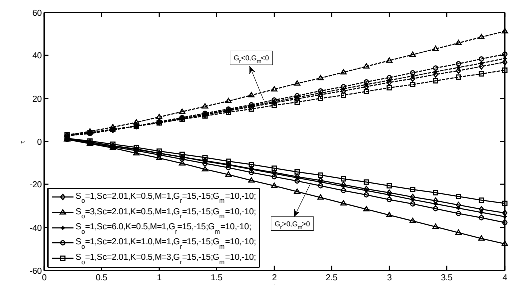

Fig. 13: Skin friction for different S_o, Sc, K and M with $\gamma = 1, R = 2, H = 2, \Pr = 0.71, \alpha = \frac{\pi}{3}$

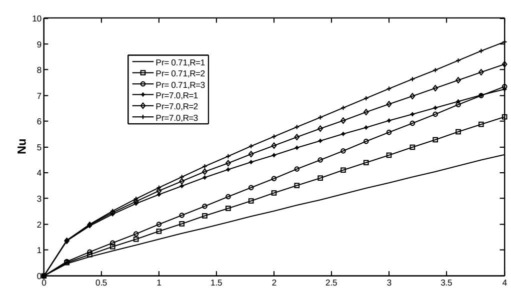

Fig. 14: Nusselt number

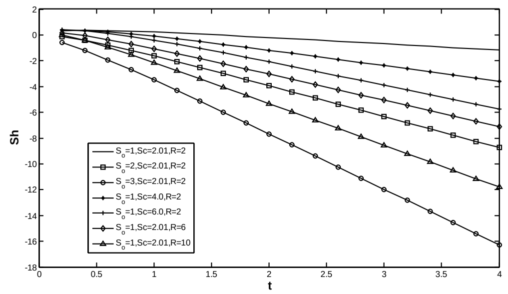

Fig. 15: Sherwood number for different S_o, Sc and R

Table 1: Velocity for different R for $G_r = 15, G_m = 10$ (cooling of the plate) with $M = 3, S_o = 5, Sc = 2.01, \Pr = 0.71, H = 4, \gamma = 1, t = 0.2, \alpha = \frac{\pi}{3}$

y	$R=2$	$R=4$	$R=6$	$R=8$
0.0	1.0000	1.0000	1.0000	1.0000
0.2	0.7894	0.7926	0.7955	0.7983
0.4	0.5644	0.5660	0.5675	0.5689
0.6	0.3646	0.3636	0.3628	0.3621
0.8	0.2140	0.2118	0.2098	0.2081
1.0	0.1147	0.1125	0.1106	0.1089
1.2	0.0564	0.0548	0.0534	0.0523
1.4	0.0255	0.0245	0.0237	0.0231
1.6	0.0106	0.0101	0.0097	0.0094
1.8	0.0041	0.0039	0.0037	0.0035
2.0	0.0015	0.0014	0.0013	0.0012

Table 2: Velocity for different R for $G_r = -15, G_m = -10$ (heating of the plate) with $M = 3, S_o = 5, Sc = 2.01, \Pr = 0.71, H = 4, \gamma = 1, t = 0.2, \alpha = \frac{\pi}{3}$

y	$R=2$	$R=4$	$R=6$	$R=8$

0.0	1.0000	1.0000	1.0000	1.0000
0.2	0.4796	0.4765	0.4735	0.4708
0.4	0.2129	0.2113	0.2098	0.2084
0.6	0.0900	0.0909	0.0917	0.0924
0.8	0.0373	0.0395	0.0415	0.0432
1.0	0.0155	0.0177	0.0196	0.0213
1.2	0.0064	0.0081	0.0094	0.0106
1.4	0.0026	0.0035	0.0043	0.0050
1.6	0.0009	0.0014	0.0019	0.0022
1.8	0.0003	0.0005	0.0007	0.0009
2.0	0.0001	0.0002	0.0003	0.0003

Table 3: Velocity for different H for $G_r = 15, G_m = 10$ (cooling of the plate) with $M = 3, S_o = 5, Sc = 2.01, \Pr = 0.71, R = 10, \gamma = 1, t = 0.2, \alpha = \frac{\pi}{3}$

y	$H = 1$	$H = 3$	$H = 5$	$H = 7$
0.0	1.0000	1.0000	1.0000	1.0000
0.2	0.7969	0.7996	0.8021	0.8045
0.4	0.5682	0.5696	0.5709	0.5721
0.6	0.3628	0.3618	0.3612	0.3607
0.8	0.2090	0.2073	0.2059	0.2047
1.0	0.1097	0.1082	0.1069	0.1057
1.2	0.0528	0.0517	0.0508	0.0500
1.4	0.0234	0.0228	0.0222	0.0216
1.6	0.0095	0.0092	0.0090	0.0087
1.8	0.0036	0.0034	0.0033	0.0032
2.0	0.0012	0.0012	0.0011	0.0011

Table 4: Velocity for different H for $G_r = -15, G_m = -10$ (heating of the plate) with $M = 3, S_o = 5, Sc = 2.01, \Pr = 0.71, R = 10, \gamma = 1, t = 0.2, \alpha = \frac{\pi}{3}$

y	$H=1$	$H=3$	$H=5$	$H=7$
0.0	1.0000	1.0000	1.0000	1.0000
0.2	0.4721	0.4695	0.4669	0.4645
0.4	0.2091	0.2077	0.2064	0.2052
0.6	0.0920	0.0927	0.0933	0.0938
0.8	0.0423	0.0440	0.0454	0.0466
1.0	0.0205	0.0220	0.0234	0.0246
1.2	0.0100	0.0111	0.0120	0.0129
1.4	0.0047	0.0053	0.0059	0.0063
1.6	0.0020	0.0024	0.0026	0.0028
1.8	0.0008	0.0009	0.0011	0.0012
2.0	0.0003	0.0003	0.0004	0.0004

Table 5: Comparison values of R for $G_r=15, G_m=10$ (cooling of the plate) with $M=3, S_o=5, Sc=2.01, \Pr=0.71, \gamma=1, H=4, t=0.2, \alpha=\frac{\pi}{2}$

	$R=2$		$R=4$	
y	Kumar [11]	Present results	Kumar [11]	Present results
0.0	1.0000	1.000000	1.0000	1.000000
0.2	0.7741	0.769195	0.7774	0.772299
0.4	0.5474	0.540788	0.5492	0.542389
0.6	0.3509	0.345260	0.3501	0.344359
0.8	0.2046	0.200826	0.2025	0.198664
1.0	0.1090	0.106896	0.1068	0.104741
1.2	0.0533	0.052256	0.0517	0.050687
1.4	0.0240	0.023522	0.0230	0.022579
1.6	0.0099	0.009766	0.0094	0.009274
1.8	0.0038	0.003745	0.0036	0.003517
2.0	0.0013	0.001328	0.0013	0.001232

5. CONCLUSIONS

An exact solution to the problem of aligned magnetic field and first order chemical reaction effects on unsteady convective heat and mass transfer flow past a vertical plate embedded in a porous medium is derived.The equations of momentum, energy and diffusion which govern the flow field are solved by the usual Laplace transform technique. The results illustrate the flow characteristics for the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number.

The main conclusions of the paper can be summarized as follows:

- The velocity decreases with an increase in the magnetic parameter and aligned angle in both cases of cooling and heating of the plate.
- The velocity increases with increasingSoret number in case of cooling of the plate, and reverse effect is observed in case of heating the plate.
- As the chemical reaction parameter increases, the fluid velocity decreasesin the case of cooling of the plate and reverse effect is observed with heating of the plate.
- An increase in the radiation parameter or heat source parameter causes a decrease in the temperature.
- The concentration decreases with an increase in the chemical reaction parameter.
- The skin-friction decreases as Soret number or Schmidt number or permeability parameter increases,where as the skin-friction increases with increasingmagnetic parameter in case of cooling of the plate and reverse effect is identified in case of heating of the plate.
- The rate of heat transfer increases due to increase in the radiation parameter orPrandtl number.
- As Soret number or Schmidt number or radiation parameter increases, the rate of mass transfer increases.

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