International Journal of Engineering & Scientific Research
Vol.5 Issue 2, February 2017,
ISSN: 2347-6532 Impact Factor: 5.900
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

RADIO D-DISTANCE NUMBER OF SOME GRAPHS

T. Nicholas*

John Bosco**

Abstract

If u, v are vertices of a connected graph *G* the *D*-length of a connected u-v path *s* is defined as $\ell^D(s) = \ell(s) + \deg(v) + \deg(u) + \sum \deg(w)$ where the sum runs over all intermediate vertices *w* of *s* and $\ell(s)$ is the length of the path. The *D*-distance $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{\ell^D(s)\}$ where the minimum is taken overall u-v paths *s* in *G*. In other words, $d^D(u, v) = \min\{\ell(s) + \deg(v) + \deg(u) + \sum \deg(w)\}$ where the sum runs over all intermediate vertices *w* in *s* and minimum is taken over all u-v paths *s* in *G*. Radio D-distance coloring is a function $f : V(G) \rightarrow \mathbb{N} \cup \{0\}$ such that $d^D(u, v) + |f(u) - f(v)| \geq diam^D(G) + 1$, where $diam^D(G)$ is the D-distance diameter of G. A D-distance radio coloring number of f is the maximum color assigned to any vertex of G. It is denoted by $rn^D(f)$. In this paper we find the radio D-distance number of some well known graphs.

Keywords:D-distance, Radio D-distance coloring, Radio D-distance number.

* **Department of Mathematics, St. Jude's College,Thoothoor, K.K.Dist, TN, India.**

** **Research Scholar, Department of Mathematics, St. Jude's College, Thoothoor, K.K.Dist, TN, India.**

1. Introduction

By a *graph* G = (V,E) we mean a finite undirected graph without loops or multiple edges. The *order* and *size* of G are denoted by p and q respectively.

Let G be a connected graph of diameter d and let k an integer such that $1 \leq k \leq d$. A radio k-coloring of G is an assignment f of colors (positive integers) to the vertices of G such that $d(u, v) + |f(u) - f(v)| \geq 1 + k$ for every two distinct vertices u, v of G. The radio k-coloring number $rc_k(f)$ of a radio k-coloring f of G is the maximum color assigned to a vertex of G. The radio k-chromatic number $rc_k(G)$ is $\min\{rc_k(f)\}$ over all radio k-colorings f of G. A radio k-coloring f of G is a minimum radio k-coloring if $rc_k(f) = rc_k(G)$. A set S of positive integers is a radio k-coloring set if the elements of S are used in a radio k-coloring of some graph G and S is a minimum radio k-coloring set if S is a radio k-coloring set of a minimum radio k-coloring of some graph G. The radio 1-chromatic number $rc_1(G)$ is then the chromatic number $\chi(G)$. When k = Diam(G), the resulting radio k-coloring is called radio coloring of G. The radio number of G is defined as the minimum span of a radio coloring of G and is denoted as rn(G).

Radio labelling (multi-level distance labelling) can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by Hale [6]. Chartrand et al.[2]. Introduced the concept of radio labeling of graph. Chartrand et al. [3] gave the upper bound for the radio number of Path. The exact value for the radio number of Path and Cycle was given by Liu, and Zhu [10]. However Chartrand et al. [2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of Cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O.Togni [8]. M.M.Rivera et al. [16] gave the radio number of $C_n \times C_n$, the cartesian product of C_n. In[4] C.Fernandez et al. found the radio number for Complete graph, Star graph, Complete Bipartite graph, Wheel graph and Gear graph. M.T.Rahim and I.Tomescu [12] investigated the radio number of Helm Graph. The radio number for the generalized prism graphs were presented by Paul Martinez et.al. in [11].

The D-distance was introduced by Reddy Babu et al. [13, 14, 15].If u, v are vertices of a connected graph *G* the *D*-length of a connected u-v path *s* is defined as $\ell^D(s) = \ell(s) + deg(v) +$

deg (u) +$\sum$ deg (w) where sum runs over all intermediate vertices w of s and $\ell(s)$ is the length of the path. The D-distance $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{\ell^D(s)\}$ where the minimum is taken overall u-v paths s in G. In other words, $d^D(u, v) = \min\{\ell(s) + \deg(v) + \deg(u) + \sum \deg(w)\}$ where the sum runs over all intermediate vertices w in s and minimum is taken over all u-v paths s in G. The D-distance eccentricity, D-distance radius and D-distance diameter are analogous to the usual path. In this paper, we introduce the concept of radio D-distance coloring. The Radio D-distance coloring is a function $f : V(G) \rightarrow \mathbb{N} \cup \{0\}$ such that $d^D(u, v) + |f(u) - f(v)| \geq diam^D(G) + 1$, where $diam^D(G)$ is the D-distance diameter of G. A radio D-distance coloring number of f is the maximum color assigned to any vertex of G. It is denoted by $rc^D(f)$. Then $rn^D(G)$ is the D-distance number of G. In this paper, we find the radio D-distance number of some well known graphs.

2. Main Result

Theorem 2.1.

For star graph $K_{1,n}$, $rn^D(K_{1,n}) \leq n+2, n \geq 2$.

Proof.

Let $V(K_{1,n}) = \{v, v_1, v_2, \ldots, v_n\}$ be vertex set, where v is the central vertex . Then $d^D(v, v_i) = n + 2, 1 \leq i \leq n$, $d^D(v_i, v_{i+1}) = n + 4, 1 \leq i \leq n - 1$, So $diam^D(K_{1,n}) = n + 4$. Define the function f as $f(v) = 0$, $f(v_i) = i + 2, 1 \leq i \leq n$. Therefore, $rn^D(K_{1,n}) \leq n + 2$.

Theorem 2.2.

For subdivision of a star graph, $rn^D(S(K_{1,n})) \leq 6n + 8, n \geq 2$.

Proof.

Let $V(S(K_{1,n})) = \{v\} \cup \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\}$ and $E(S(K_{1,n})) = \{e_1, e_2, \ldots, e_n\} \cup \{s_1, s_2, s_3, \ldots, s_n\}$. Consider v is the center vertex then v is adjacent to $\{u_1, u_2, \ldots, u_n\}$ and $\{u_1, u_2, \ldots, u_n\}$ are adjacent to $\{v_1, v_2, \ldots, v_n\}$. Then $d^D(v, u_i) = n + 3$, $d^D(v, v_i) = n + 5, 1 \leq i \leq n$, if u_i and v_j are adjacent $d^D(u_i, v_j) = 4$, if u_i and v_j are not adjacent $d^D(u_i, v_j) = n + 8$, $d^D(v_i, v_{i+1}) = n$

+ 10, So $diam^D(S(K_{1,n})) = n + 10$. Define the function f as $f(v) = 0$, $f(v_i) = n + 4$, $1 \leq i \leq n$, $f(u_i) = n + 5i + 8$, $1 \leq i \leq n$. Therefore, $rn^D(S(K_{1,n})) \leq 6n + 8$.

Theorem 2.3.

For complete graph K_n, $rn^D(K_n) = n - 1$, $n \geq 2$.

Proof.

Since $diam^D(G) = d^D(u, v)$ for any u, v ∈ $V(K_n)$ using radio D-distance implies $|f(u) - f(v)| \geq 1$ for all u, v ∈ $V(K_n)$. Since $f : V(K_n) \to \mathbb{N} \cup \{0\}$ is injective it follows that $rn^D(K_n) \leq n - 1$. Since $|V| = n$, $rn^D(K_n) \geq n - 1$. Hence the result.

Theorem 2.4.

For complete bipartite $K_{m,n}$, $rn^D(K_{m,n}) \leq 2n + m$ if $n \geq 3, m \geq 2$.

Proof

Let $\{v_1, v_2, v_3, \ldots, v_m\}$ and $\{u_1, u_2, u_3, \ldots, u_n\}$ be the partite sets. Then if v_i and u_j are adjacent $d^D(v_i, u_j) = n + m + 1$, $d^D(v_i, v_{i+1}) = 2n + m + 2$, $d^D(u_j, u_{j+1}) = n + 2m + 2$, So $diam^D(G) = 2(n + 1) + m$. Define the function f as $f(v_i) = i - 1$, $1 \leq i \leq m$ $f(u_i) = m + n + i$, $1 \leq i \leq n$. Therefore , $rn^D(K_{m,n}) \leq 2n + m$.

Note. When m = n, $rn^D(K_{m,n}) \leq 3n$.

❖ The graph $C_n^{(t)}$denoting the one point union of t copies cycle C_n. The graph $C_3^{(t)}$(or $K_3^{(t)}$) is called friendship graph.

Theorem 2.5.

For friendship graph $C_3^{(t)}$, $rn^D(C_3^{(t)}) \leq 3t + 5$, $t \geq 2$

Proof:

Let $V(G) = \{v, v_1, v_2, \ldots, v_{2t}\}$ be the vertex set, where v is the central vertex. Then $d^D(v, v_i) = 2t + 3$, $1 \leq i \leq 2t$, if v_i and v_{i+1} are adjacent $d^D(v_i, v_{i+1}) = 5$, $1 \leq i \leq 2t - 1$, if v_i and v_{i+1} are not adjacent $d^D(v_i, v_{i+1}) = 2t + 6$, $1 \leq i \leq 2t - 1$, So $diam^D(G) = 2t + 6$. Define the function f as $f(v) = 0$, if i is odd then $f(v_i) = \left(\frac{i-1}{2}\right) + 4$ and if i is even then $f(v_i) = 2t + \left(\frac{i}{2}\right) + 5$. Therefore, $rn^D(C_3^{(t)}) \leq 3t + 5$.

Reference

[1] F. Buckley and F. Harary, *Distance in Graphs*,Addition- Wesley, Redwood City, CA, 1990.

[2] G. Chartrand, D. Erwinn, F. Harary, and P. Zhang, “Radio labeling of graphs,” *Bulletin of the Institute of Combinatorics and Its Applications*, vol. 33, pp. 77–85, 2001.

[3] G. Chartrand, D. Erwin, and P. Zhang, Graph labeling problem suggested by FM channel restrictions, Bull. Inst. Combin. Appl., 43, 43-57(2005).

[4] C. Fernandaz, A. Flores, M. Tomova, and C. Wyels, The Radio Number of Gear Graphs, arXiv:0809. 2623, September 15, (2008).

[5] J.A. Gallian, A dynamic survey of graph labeling, Electron. J. Combin. 19 (2012) #Ds6.

[6] W.K. Hale, *Frequency assignment: Theory and applications*, Proc. IEEE 68 (1980), pp. 1497–1514.

[7]F. Harary, Graph Theory, Addision wesley, New Delhi (1969).

[8] R. Khennoufa and O. Togni, The Radio Antipodal and Radio Numbers of the Hypercube, ccepted in 2008 publication in ArsCombinatoria.

[9] D. Liu, Radio number for trees, Discrete Math. 308 (7) (2008) 1153–1164.

[10] D. Liu, X. Zhu, Multilevel distance labelings for paths and cycles, SIAM J. Discrete Math. 19 (3) (2005) 610–621.

[11] P. Murtinez, J. OrtiZ, M. Tomova, andC. Wyles, Radio Numbers For Generalized Prism Graphs, Kodai *Math. J.*, **22**,131-139(1999).

[12] M. T. Rahim, I. Tomescu, OnMulti-level distance labelings of Helm Graphs, accepted for publication in ArsCombinatoria.

[13] Reddy Babu,D., Varma, P.L.N.,*D-distance in graphs*, Golden ResearchThoughts, 2 (2013),53-58.

[14] Reddy Babu, D., Varma, P.L.N.,Average *D*-Distance Between Vertices Of A Graph , Italian Journal Of Pure And Applied Mathematics - N. 33¡2014 (293¡298).

[15] Reddy Babu, D., Varma, P.L.N., Average *D*-Distance Between Edges Of A Graph ,Indian Journal of Science and Technology, *Vol 8(2), 152–156, January 2015.*

[1 6] M. M. Rivera, M. Tomova, C. Wyels, and A. Yeager, The Radio Number of Cn _Cn,re submitted to Ars Combinatoria, 2009.