

HYPOTHESIS TESTING: AN EXPLANATION

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ABSTRACT

This paper deals with a statistical method 'hypothesis testing'. In the methodology of hypothesis testing, required information is gathered from a random sample collected from a certain population which requires to be analyzed. This 'sample' information is subjected to suitable statistical investigation and assessment after which, the results are cautiously extended to generalize about the population from which the sample was gathered. Hence, hypothesis testing is a basic and essential process to make a suitable inferential decision about population of interest to the researcher.

Hypothesis testing, therefore, is to be understood as an essential and necessary procedure in statistics. A hypothesis test weighs and assesses two mutually exclusive statements about a population under examination in order to determine which of these two statements are best described and supported by the sample data. When it is inferred about the sample data that it is 'statistically significant', we want to actually know what exactly this signifies about the population. We also are interested in knowing how these tests work practically and what their usefulness is in real world and further, what does 'statistical significance' actually mean?

The paper would help to intuitively understand how hypotheses tests are applied in practical situations by focusing more on concepts rather than on formulae, equations and numbers.

KEY WORDS : Statistical Significance, Null Hypothesis, Alternate Hypothesis, Population, Sample, Inference, Probability, Level of Confidence, Region of Rejection, Central Limit Theorem

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INTRODUCTION

Testing of hypotheses is generally and largely found to be rather confusing and a stumbling block to many researchers. Researchers look upon hypotheses testing as something which is intriguing, vague and difficult to comprehend, interpret as well as apply. In fact, in the minds of several researchers, there arise puzzling circumstances where the researcher is at crossroads about why and how to apply hypotheses testing and most important, how to practically analyze the results obtained.

A hypothesis, in statistics, is a statement of assumption made about a population. Generally, this statement of assumption is specified with a numerical value. In testing this assumption made, we gather data in an effort to garner evidence in support of our assumption made about the population under consideration.

There are certain steps that are required to be followed while conducting a test (or, analysis) regarding the hypothesis. These steps are explained below.

Step One:

Each hypothesis test takes into consideration *two hypotheses* about the population under consideration. One is termed as the null (or, unbiased, or unprejudiced) hypothesis which is, generally, a statement of a particular *parameter* value. This hypothesis is assumed to be true until there is evidence to suggest otherwise. The second hypothesis is generally termed as the alternative hypothesis. This *alternative* hypothesis is a statement of assumption regarding a range of alternative values in which the parameter of the null hypothesis may fall.

Step Two:

We have to set up a level of confidence, or, a level of significance, when we are testing our assumption (or, hypothesis): To explain this, let us consider a salesman. When he ventures to sell his products in the market, he has a certain level of confidence in him, and this is generally high, around 90%, or 95%, or 99%. This is because, no salesman would venture to sell his products/services if his confidence is only, say, 40%. Again, we never speak of a confidence level in terms of 92.23%, or, 94.6%, etc. Question also arises, 'why not 100% confidence?' There is a risk involved in this, that is, there can be several unknown errors that may affect the sales. In a

way, therefore, the salesman has left for himself *no chance to wriggle himself out* in case he does not meet the 100% level.

Next, when the salesman says he's 90% confidence, he indicates that the chances of his making an error are possibly 10%. This 'probability of error occurring', may **significantly** affect his sales. Similar argument holds true in the case of hypotheses testing wherein the level of significance, or, the chances, or, probability of making an error are either, 10%, or, 5%, or, 1%, as the case maybe. This probability of error that significantly affects our original assumption, is denoted by the Greek symbol, α (alpha)

Step Three:

From the sample data collected, a suitable *test-statistic* has to be calculated, e.g. , sample mean, sample standard deviation, chi-square, or any other test-statistic which can be logically used to compare these values obtained from the sample, with the population parameter concerned, in order to draw inferences and conclusions about the population. The test-statistic is calculated under the *basic assumption* that the null hypothesis is *true*.

Quite obviously, if we assume that null hypothesis is false *from the beginning*, there is then no need to *test* anything!). Assumptions are also made regarding the distribution of sample data (for applying the Central Limit Theorem), dependency/independency of sample values, etc. All this is done so that the researcher is specific and clear regarding the domain or framework within which his analysis is being carried out.

Step Four:

A suitable, appropriate method (or, an expression) is now used to calculate either the *p-value* (probability value) or the *z-value/t-value/rejection region*. All these calculations are done only to see if our values calculated from the sample, fall either in the region of acceptance or in the region of rejection at the pre-determined critical values (or, the *cut-off* values). These depend on the levels

of confidence at which we are working, namely, 90%, 95%, 99% (or, levels of significance of 10%, 5%, 1%). The region of rejection is *more extreme* than the critical value. Critical value is determined whether we are working on a one-tail test or a two-tail test as well as on the level of confidence (or, level of significance)

Step Five:

Depending upon the *observation* that we make in **Step Four** above, we have now to decide (that is,

to infer) whether we are going to either *reject* the null hypothesis or, *fail to reject* the null hypothesis. All our

inferences, therefore, are strictly based on our sample chosen. Further, in case it so happens that we have to accept the null hypothesis, we do not *infer any decision*, but merely state that, ‘as per the sample taken, there seems to be no reasonable evidence *not to accept* the null hypothesis’.

Step Six:

This final step is drawing our overall conclusion about the population under study, based upon our above calculations. Since we are analyzing a sampled data and drawing a conclusion from it, we have to be cautious while passing *judgment* (that is, making a conclusion). In case we infer to reject the null hypothesis, we have to *preferably* state the conclusion as follows: *as per the sample data collected and analyzed, it appears that*

there seems to be reasonable grounds to conclude that the population parameter does not appear to be the true representative of the population. This is because, if another researcher performs the same research with the help of a new sampled data about the same population, he may possibly arrive at a conclusion which could result in *acceptance* of the null hypothesis!

The classic examples could possibly be the opinion polls conducted just before elections. One can literally see how different predictions are made by pollsters, in terms of analysis of sampling data.

SCENARIO ONE

Let us start by considering the Opinion Poll conducted *before* any election. It is evident here that the number of eligible voters is extremely large in any given constituency and for a researcher, to approach *each and every eligible* voter to seek his views and opinion, is practically impossible. Instead, a researcher prefers to study a *sample* of voters obtained from the population of the voters (that is, the total number of eligible voters) in a particular constituency.

The researcher now collects and records the views and opinion of each of the voters in the sample collected. He then analysis this sample data. Then, based upon his findings, *he infers and projects* about the possible opinion and the views of the entire population of voters under consideration. What is being done here is that the researchers *assumes* that the sample under consideration is a *possibly a true representative* of the entire population of voters in the constituency. Common sense dictates that this *need not necessarily* be so because the distribution may be skewed, even though for the purposes of analysis we assume the prevalence of *normality* as per the Central Limit Theorem.

Since the research regarding the opinion of the voters is being carried out *before the elections* take place, the researcher assumes that what he is resorting to is possibly correct, he therefore starts with an assumption such as: 'I am going to conduct a research about the opinion/views of a sample of voters regarding their voting pattern with a confidence of 90% (or, 95%, or 99%)'. In other words, the researcher is carrying out his research in an unbiased and in an unprejudiced manner. This initial assumption made by the researcher is called as an *unbiased assumption*, or, *unprejudiced assumption*, or, *null hypothesis*. If, instead, the researcher is going to go about this research with bias and prejudice, the very purpose of research will be *defeated*. It is similar to the case of a judge in a High Court who is already prejudiced to, for example, pronounce the accused as guilty, the very purpose of conducting the trial would be defeated.

The concept of working with a confidence of 90%/95%/99% is as follows. No researcher would interested in conducting a research with a low confidence of, say, 50%, or, 40%, etc. The purpose of such a research would be '*nipped-in-the-bud*' itself, as it would be futile, meaningless

and serve no purpose. It would be akin to launching a marketing or sales strategy with, say, only 50% confidence! This would, in all practicality, never take off!

After the elections are over and the results are declared, the researcher *may* find that **(a)** that what he had predicted about the entire population of voters based on his analysis from the sample of voters turned out to be *correct* (or, true), or, **(b)** the prediction made about the population of voters based on the analysis from the sample of voters, turned out to be *incorrect* (or, false).

If prediction **(a)** turned out to be correct, the researcher concludes, or infers, that what he has concluded from the sample analysis has been true. If instead, **(b)** had occurred, the researcher would then conclude that his analysis and conclusion, based on the sample, was incorrect. He then states that an error has occurred in his analysis and accepts this fact. This is termed as an *alternate judgment* passed about his original assumption about the sample, which he had conducted either with 90%, or 95%, or 99%, confidence.

SCENARIO TWO

Let us consider the case of 'instant coffee' manufacturers. They pack the coffee in sachets of various weights, namely, 50 gm, 100 gm, etc. Let us assume that you are consumer activist who is keen to find out whether, say, the 50 gm sachet *actually* (or, *really*) contains 50 gm (with a permissible and acceptable error within certain limits), or it contains some other quantity (by weight) which may either be *appreciably and significantly more*, or *appreciably and significantly less*, than 50 gm. Now, in case you decide to go to the manufacturer concerned and tell him that you *know definitely* that the sachet contains amount *much different* than 50 gm, it is obvious that *you are already biased and prejudiced*. The need, therefore, to conduct *any test* does not arise since you have *already concluded a judgment*.

Now, instead, let us consider the following situation. The manufacturer informs you that they have no reason to doubt that all their sachets contain, on an average, 50 gm of instant coffee. That is, the *population of sachets all contain, on an average (that is, within permissible limits of error)*, an amount of 50 gm of instant coffee.

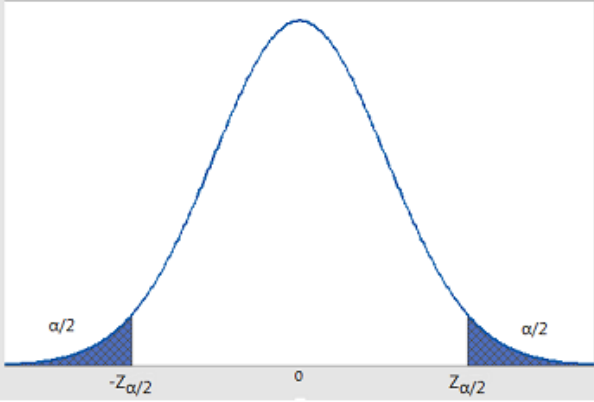
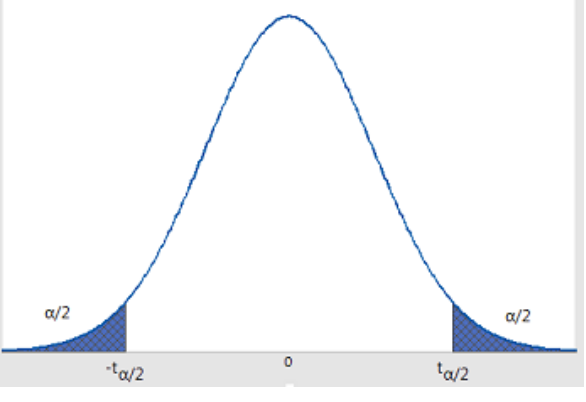
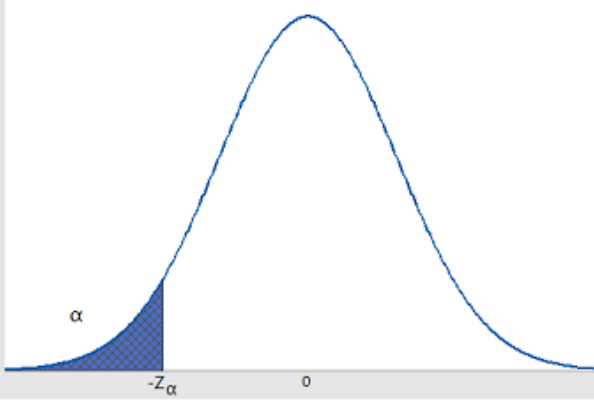
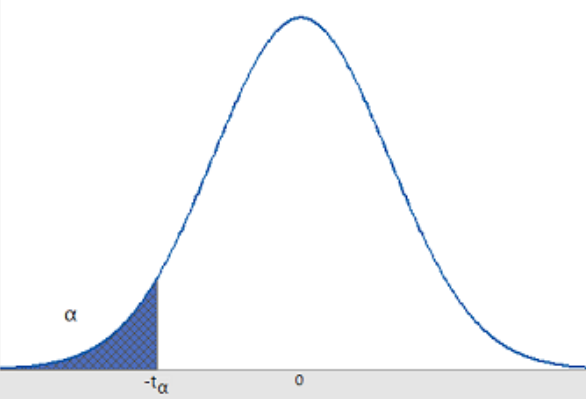
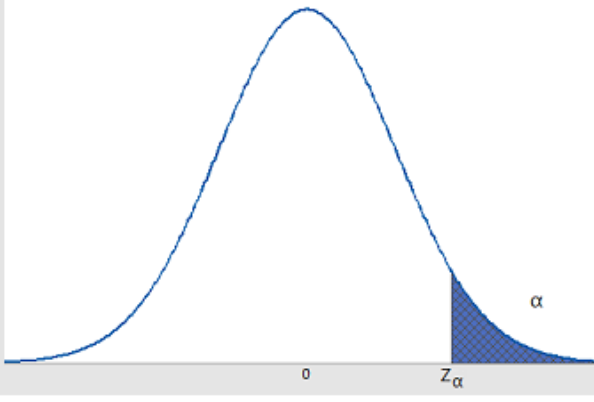
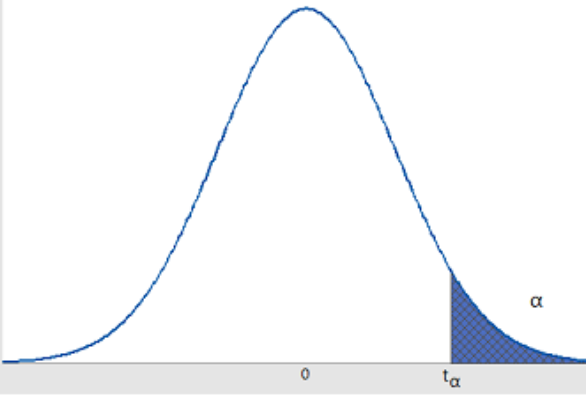
You, as a consumer activist, wish to test this *claim* being made by the manufacturer. But, in all fairness to the manufacturer, you start with the following assumption which may be stated as follows: '*I have no evidence at present and hence, no reason to doubt the claim made by the manufacturer that their sachets contain 50 gm of instant coffee on an average*'. This is an unbiased or unprejudiced way of starting an investigation. This is called as '*null hypotheses*'. *Symbolically*, it is written as follows: $H_0: \mu = 50 \text{ gm}$. Hence, this mathematical statement of null hypothesis is only an *abbreviated* way of writing the long statement written in several words. That's all.

As a consumer activist, you would now go about collecting samples of sachets in a random and unbiased manner. To make your inferences and conclusions reasonably true and acceptable, you will collect at least, say, a thousand sachets, empty the contents from each sachet, and weigh the contents of each sachet very accurately. You may then find the *arithmetic mean* of all the weights so obtained as well as the *standard error*. Next, you will calculate the suitable test-statistic and find the corresponding *p-value*.

If the *calculated p-value* falls within the region of acceptance, you will infer and conclude that you have, as per the sample analysis, no reason to doubt the claim being made by the company. That is, you accept the null hypothesis. Mathematically stated: *Accept* $H_0: \mu = 50 \text{ gm}$. Hence, for *lack of conclusive evidence to the contrary*, you accept the claim made by the manufacturer or, you have no reason to doubt the claim being made by the manufacturer. We use this language, or wording, in our conclusion because, in the future, some other activist may take a fresh sample and possibly prove the manufacturer's claim to be false.

REJECTION REGION APPROACH TO HYPOTHESIS TESTING

We first establish the appropriate critical values for the tests using the Z-table for test of one proportion, or the t-table if a test for one mean. We then write down clearly the rejection region for the problem.

One Proportion Z-test	One Mean <i>t</i> -test
 <p>Two-Tailed Reject H0 if $Z^* \geq Z_{\alpha/2}$ $Z^* \geq Z_{\alpha/2}$</p>	 <p>Two-Tailed Reject H0 if $t^* \geq t_{\alpha/2}$ $t^* \geq t_{\alpha/2}$</p>
 <p>Left-Tailed Reject H0 if $Z^* \leq -Z_{\alpha}$ $Z^* \leq -Z_{\alpha}$</p>	 <p>Left-Tailed Reject H0 if $t^* \leq -t_{\alpha}$ $t^* \leq -t_{\alpha}$</p>
 <p>Right-Tailed Reject H0 if $Z^* \geq Z_{\alpha}$ $Z^* \geq Z_{\alpha}$</p>	 <p>Right-Tailed Reject H0 if $t^* \geq t_{\alpha}$ $t^* \geq t_{\alpha}$</p>

Now, check whether the value of the test statistic falls/does not fall in the rejection region. If it does, then **reject** null hypothesis. If it does not fall in the rejection region, **do not reject** null hypothesis.

P-VALUE APPROACH TO HYPOTHESIS TESTING

We first compute the appropriate p -value based on our **alternative** hypothesis:

If our H_a is right-tailed, then the p -value is the probability the sample data produces a value **equal to or greater** than the observed test statistic.

If our H_a is left-tailed, then the p -value is the probability the sample data produces a value **equal to or less** than the observed test statistic.

If our H_a is two-tailed, then the p -value is **two times** the probability the sample data produces a value **equal to or greater** than the **absolute value** of the observed test statistic.

<i>Right-tailed</i>		<i>Left-tailed</i>		<i>Two-tailed</i>
$P(Z > Z^*)P(Z > Z^*)$	OR	$P(Z < Z^*)P(Z < Z^*)$	OR	$2 \times P(Z > Z^*)2 \times P(Z > Z^*)$
$\backslash(P(t > t^*)) \text{ at } df = n-1$		$P(t < t^*)P(t < t^*) \text{ at } df = n-1$		$2 \times P(t > t^*)2 \times P(t > t^*) \text{ at } df = n-1$

Now we have to check to see if the p -value is **less than** the stated alpha value. If it is, then reject null hypothesis. If it is **not less than alpha**, **do not reject** the null hypothesis

CONCLUSION

From what has been explained above, we conclude that a research hypothesis is basically an assertion or a statement made by a researcher when he deliberates upon the outcome of a research or experiment

The hypothesis is, therefore, created with the help of a number of different methods and approaches. But largely speaking, it is usually the result of a process of inductive reasoning where observations made from a selected sample data lead to the formation of a theory.

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