International Journal of Management, IT & Engineering
Vol. 7, Issue 6, June- 2017,
ISSN: 2249-0558 Impact Factor: 7.119
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gate as well as in Cabell's Directories of Publishing Opportunities, U.S.A

APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS IN FLUID MECHANICS

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ABSTRACT

Partial differential equations (PDEs) develop in all fields of building and science. Most genuine physical procedures are spoken to by partial differential equations. A partial differential equation (PDE) is an equation expressing a connection between a component of at least two independent variables and the partial subsidiaries of this capacity as for these independent variables. The dependent variable f is utilized as a nonexclusive ward variable. In most problems in engineering and science, the independent variables are either space (x, y, z) or space and time (x, y, z, t). The current paper highlights the applications of partial differential equations in fluid mechanics.

KEYWORDS:
Differential equations, Fluid, Variable

INTRODUCTION

The dependent variable depends on the physical problem being modeled. Examples of three simple partial differential equations having two independent variables are presented below:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \tag{1}$$

$$\frac{\partial f}{\partial t} = \alpha \frac{\partial^2 f}{\partial x^2} \tag{2}$$

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2} \tag{3}$$

Equation (1) is the two-dimensional *Laplace equation*, Eq. (2) is the one-dimensional *diffusion equation*, and Eq. (3) is the one-dimensional *wave equation*. For simplicity of notation, Eqs. (1) to (3) usually will be written as

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$$f_{xx} + f_{yy} = 0 \quad (4)$$

$$f_t = \alpha f_{xx} \quad (5)$$

$$f_{tt} = c^2 f_{xx} \quad (6)$$

where the subscripts denote partial differentiation.

The solution of a partial differential equation is that particular function, $f(x, y)$ or $f(x, t)$, which satisfies the PDE in the domain of interest, $D(x, y)$ or $D(x, t)$, respectively, and satisfies the initial and/or boundary conditions specified on the boundaries of the domain of interest. In a very few special cases, the solution of a PDE can be expressed in closed form. In the majority of problems in engineering and science, the solution must be obtained by numerical methods.

Equations (4) to (6) are examples of partial differential equations in two independent variables, x and y, or x and t. Equation (4), which is the two-dimensional Laplace equation, in three independent variables is

$$\nabla^2 f = f_{xx} + f_{yy} + f_{zz} = 0 \quad (7)$$

where ∇^2 is the Laplacian operator, which in Cartesian coordinates is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (8)$$

Equation (5), which is the one-dimensional diffusion equation, in four independent variables is

$$f_t = \alpha(f_{xx} + f_{yy} + f_{zz}) = \alpha \nabla^2 f \quad (9)$$

The parameter α is the diffusion coefficient. Equation (6), which is the one-dimensional wave equation, in four independent variables is

$$f_{tt} = c^2(f_{xx} + f_{yy} + f_{zz}) = c^2 \nabla^2 f \quad (10)$$

The parameter c is the wave propagation speed. Problems in two, three, and four independent variables occur throughout engineering and science.

Equations (III.4) to (III. 10) are all *second-order* partial differential equations. The order of a PDE is determined by the highest-order derivative appearing in the equation. A large number of physical problems are governed by second-order PDEs. Some physical problems are governed by a first-order PDE of the form

$$af_t + bf_x = 0 \quad (11)$$

where a and b are constants. Other physical problems are governed by fourth-order PDEs such as

$$f_{xxxx} + f_{xxyy} + f_{yyyy} = 0 \quad (12)$$

Equations (4) to (12) are all *linear* partial differential equations. A linear PDE is one in which all of the partial derivatives appear in linear form and none of the coefficients depends on the dependent variable.

APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS IN FLUID MECHANICS

We consider a string of length L stretched out along the x-axis, one end of the string being at $x = 0$ and the other being at $x = L$. We assume that the string is free to move only in the vertical direction. Let $u(x, t)$ = vertical displacement of the string at the point x at time t.

We will derive a partial differential equation for $u(x, t)$. Note that since the ends of the string are fixed, we must have $u(0,t) = 0 = u(L,t)$ for all t.

It will be convenient to use the "configuration space" V_0. An element $u(x) \in V_0$ represents a configuration of the string at some instant of time. We will assume that the *potential energy* in the string when it is in the configuration $u(x)$ is

$$V(u(x)) = \int_0^L \frac{T}{2}\left(\frac{du}{dx}\right)^2 dx,$$

where T is a constant, called the *tension* of the string.

Indeed, we could imagine that we have devised an experiment that measures the potential energy in the string in various configurations, and has determined that does indeed represent the total potential energy in the string. On the other hand, this expression for potential energy is quite plausible for the following reason: We could imagine first that the amount of energy in the string should be proportional to the amount of stretching of the string, or in other words, proportional to the length of the string. From vector calculus, we know that the length of the curve $u = u(x)$ is given by the formula

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Length $\int_0^L \sqrt{1+(du/dx)^2}dx.$

But when du/dx is small,

$$\left[1+\frac{1}{2}\left(\frac{du}{dx}\right)^2\right]^2 = 1+\left(\frac{du}{dx}\right)^2 + \text{a small error}$$

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and hence

$\sqrt{1+(du/dx)^2}$ is closely approximated by $1+\frac{1}{2}(du/dx)^2$

Thus to a first order of approximation, the amount of energy in the string should be proportional to

$$\int_0^L \left[1+\frac{1}{2}\left(\frac{du}{dx}\right)^2\right] dx = \int_0^L \frac{1}{2}\left(\frac{du}{dx}\right)^2 dx + \text{constant.}$$

Letting T denote the constant of proportionality yields

energy in string = $\int_0^L \frac{T}{2}\left(\frac{du}{dx}\right)^2 dx + \text{constant}$

Potential energy is only defined up to addition of a constant, so we can drop the constant term to obtain.

The force acting on a portion of the string when it is in the configuration $u(x)$ is determined by an element $F(x)$ of V_0. We imagine that the force acting 011 the portion of the string from x to $x+dx$ is $F(x)dx$. When the force pushes the string through an infinitesimal displacement $\xi(x) \in V_0$, the total *work* performed by $F(x)$ is then the "sum" of the forces acting on the tiny pieces of the string, in other words, the work is the "inner product" of F and ξ,

$$\langle F(x), \xi(x) \rangle = \int_0^L F(x)\xi(x)dx$$

On the other hand this work is the amount of potential energy lost when the string undergoes the displacement:

$$\langle F(x), \xi(x) \rangle = \int_0^L \frac{T}{2}\left(\frac{\partial u}{\partial x}\right)^2 dx - \int_0^L \frac{T}{2}\left(\frac{\partial (u+\xi)}{\partial x}\right)^2 dx$$

$$= -T\int_0^L \frac{\partial u}{\partial x}\frac{\partial \xi}{\partial x}dx + \int_0^L \frac{T}{2}\left(\frac{\partial \xi}{\partial x}\right)^2 dx.$$

We are imagining that the displacement ξ is infinitesimally small, so terms containing the square of ξ or the square of a derivative of ξ can be ignored, and hence

$$\langle F(x), \xi(x) \rangle = -T\int_0^L \frac{\partial u}{\partial x}\frac{\partial \xi}{\partial x}dx$$

Integration by parts yields

$$\langle F(x), \xi(x) \rangle = T\int_0^L \frac{\partial^2 u}{\partial x^2}\xi(x)dx - T\left(\frac{\partial u}{\partial x}\xi\right)(L) - T\left(\frac{\partial u}{\partial x}\xi\right)(0)$$

Since. $\xi(0) = \xi(L) = 0$ we conclude that

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$$\int_0^L F(x)\xi(x)dx = \langle F(x), \xi(x) \rangle = T\int_0^L \frac{\partial^2 u}{\partial x^2}\xi(x)dx$$

Since this formula holds for *all* infinitesimal displacements $\xi(x)$, we must have

$$F(x) = T\frac{\partial^2 u}{\partial x^2}$$

for the force density per unit length.

CONCLUSION

Fluids can be further subcatergorized. There are perfect or inviscid fluids. In such fluids, the main inside power display is weight which acts with the goal that fluid flows from a district of high weight to one of low weight. The equations for a perfect fluid have been connected to wing and airplane design (as a farthest point of high Reynolds number flow). However fluids can show inward frictional powers which model a stickiness" property of the fluid which includes vitality loss; these are known as thick fluids. A few fluids/material known as \non-Newtonian or complex fluids" display considerably more bizarre conduct, their response to misshapening may rely upon: (i) previous history (prior distortions), for instance a few paints; (ii) temperature, for instance a few polymers or glass; (iii) the measure of the disfigurement, for instance a few plastics or silly putty.

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