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Pre-open set in the topological spaces

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Abstract : In this paper, we introduce a new class of set in the topological space, namely Pre-open set in the topological space. We find characterizations of these sets. Further, we study some fundamental properties of these sets in the topological space.

Keywords : open set, closed set, interior, closure, Pre-open set

I. Introduction

The term Pre-open set is called nearly open set and locally dense. Pre-open set plays a significant role in the topological space. Ever since the concept of the term Pre-open set in a topological space was first introduced by the three mathematician Mashhour, Abd. El-Monsef & El-Deeb [12] in the year 1982. In the topological space, the repeated application of interior and closure operators leads to several different new classes of sets. Some of them are generalized form of an open sets. While few others are the so called Pre-open set. The authoress is not in a position to trace out the original source of the concept of Pre-open set but it is proposed to make a systematic study of these concepts in the following sections of this chapter.

In this paper, analogous to Mashhour, Abd. El-Monsef & El-Deeb [12]'s Pre-open set, we try to attempt to develop certain results based on these set in the topological space.

II. Preliminaries

Throughout this paper (X , τ) is always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X , τ) then $C_L(A)$ & $I_N(A)$ are denote the closure and interior of the set A in the topological space.

2.1 . Pre-open set[12] : Let (X , J) be a topological space of a nonempty set X with the topology J & let $A \subseteq X$ then the set A is said to be Pre-open set if and only if $A \subseteq I_N[C_L(A)]$.

That is "a Pre-open set is a set which is contained in interior of its closure".

The family of all Pre-open set in the topological space is denoted by F[PO(X)].

2.2 . Proposition : The set of real space R with the usual topology J is always a Pre-open set.

We shall show it by taking an example.

Let X = R ; (the real space) and $C_L(R) = R$ and $I_N\{C_L(R)\} = I_N\{R\} = R$.

So, $I_N\{C_L(R)\} = R$ and $R \subseteq R$. $\Rightarrow$ **Hence, $X \subseteq I_N\{C_L(X)\}$.**

Therefore, the set of real space R with the usual topology J is always a Pre-open set in the topological space.

2.3 . Proposition : The empty set φ is always a Pre-open set in the real space with the usual topology.

We shall show it by taking an example in the real space R.

Let X = φ ; (the empty set) and $C_L(\varphi) = \varphi$ and $I_N\{C_L(\varphi)\} = I_N\{\varphi\} = \varphi$.

So, $I_N\{C_L(\varphi)\} = \varphi$ & $\varphi \subseteq \varphi$ **Hence, $X \subseteq I_N\{C_L(X)\}$.**

Therefore, the set φ is always a Pre-open set in the real space with the usual topology.

2.4 . Proposition : Every open set is always a Pre-open set in the real space with the usual topology.

We shall show it by taking an example in the real space R with the usual topology.

Let X =]a , b[; (open interval)

Then $C_L(\,]a,b[\,) = [a,b]$ & $I_N\{C_L(\,]a,b[\,)\} = I_N\{[a,b]\} = \,]a,b[$.

So, $I_N\{C_L(\,]a,b[\,)\} = \,]a,b[$ & $]a,b[\subseteq \,]a,b[$. **Hence, $X \subseteq I_N\{C_L(X)\}$.**

Therefore, every open interval is always a Pre-open set in the real space with the usual topology. As we know that as every open interval is an open set in the topological space.

Hence, every open set is always a Pre-open set in the real space with the usual topology.

2.5 . Proposition : Every closed set is not a Pre-open set in the real space with the usual topology.

We shall show it by taking an example in the real space R with the usual topology .

Let X = [a , b] ; (closed interval)

Then C_L ([a , b]) = [a , b] & I_N {C_L([a , b])} = I_N {[a , b]} =]a , b[. But, [a , b] $\nsubseteq$]a , b[

$\Longrightarrow$ X $\nsubseteq$ I_N{C_L ([a , b])}. **Hence, X $\nsubseteq$ I_N{C_L (X)}.**

So, every closed interval is not a Pre-open set in the real space with the usual topology.

As we know that every closed interval is always a closed set in the real space.

Thus, every closed set is not a Pre-open set in the real space with the usual topology.

2.6 . Proposition : Every half open interval is not a Pre-open set in the real space with the usual topology.

We shall show it by taking an example in the real space R with the usual topology.

Case – [1] : Let X = [a , b [,

Let X = [a , b [; (half open interval)

So, C_L{ [a , b[} = [a , b] and I_N{C_L([a , b[ )} = I_N{[a , b]} =]a , b[.

But, [a , b[$\nsubseteq$]a , b[$\Longrightarrow$ [a , b[$\nsubseteq$ I_N{C_L([a , b[ )}. **Hence, X $\nsubseteq$ I_N { C_L (X) }.**

Thus, every half open interval X = [a , b [is not a Pre-open set in the real space with the usual topology.

Case – [2] : Let X =] a , b] ,

Let X =] a , b] ; (half open interval)

So, $C_L($]a , b]) = [a , b] and $I_N\{C_L($]a , b])} = I_N\{ [a , b] \} =]a , b[.

But,]a , b] ⊈]a , b[⟹] a , b] ⊈ I_N {C_L([a , b[ )}.

Hence, X ⊈ I_N\{ C_L (X) \}.

Therefore, every half open interval X =] a , b] is not a Pre-open set in the real space with the usual topology.

2.7 . Proposition : The union of two Pre-open set is a Pre-open set in the topological space.

We shall show it by taking an example in the real space R with the usual topology.

We have discussed earlier that the every open interval is the Pre-open set.

Let F =]a , b[U]b , c[; where, a < b < c .

So, F =]a , b[U]b , c[=]a , c[- {b} & C_L (F) = C_L [{]a , c[- {b}}] = [a , c] &

I_N [{C_L (F)}] = I_N [{ [a , c] }] =]a , c [&]a , c[- {b} ⊆]a , c [; a < b < c.

Hence, F ⊆ I_N [{C_L (F)}].

Therefore, the union of two Pre-open set that is F =] a , b [U] b , c [is also a Pre-open set in the topological space.

2.8 . Proposition :The intersection of two Pre-open set is always a Pre-open set in the topological space.

We shall show it by taking an example in the real space R with the usual topology.

We have discussed earlier that the every open interval is the Pre-open set.

Let F =] a , b [∩] b , c [; where, a < b < c .

So, F =] a , b [∩] b , c [= φ & C_L(F) = C_L [{φ}] = φ & I_N [{C_L (F)}] = I_N [{φ}] = φ

& φ ⊆ φ. So, φ ⊆ I_N [C_L{(F)}].

Hence, F ⊆ I_N [C_L {(F)}].

Thus, the intersection of two Pre-open set that is $F =]a , b[\cap]b , c[$; $a < b < c$ is a Pre-open set in the topological space.

2.9 . Proposition : The intersection of arbitrary collection of Pre-open set is not a Pre-open set in the topological space.

Proof : Let us consider, $A_n =] -\frac{1}{n}, \frac{1}{n} [\ \forall n \in N$ then

$A_1 =] -1 , 1[$, $A_2 =] -\frac{1}{2}, \frac{1}{2} [$, $A_3 =] -\frac{1}{3}, \frac{1}{3} [$, $A_4 =] -\frac{1}{4}, \frac{1}{4} [$ ………..

Since, every open interval is an open set and also every open set is Pre-open set.

Therefore, $\{ A_n ; \forall n \in N \}$ is a family of an infinite number of Pre-open set.

Clearly, $A_1 \cap A_2 =] -1 , 1[\cap] -\frac{1}{2}, \frac{1}{2} [=] -\frac{1}{2}, \frac{1}{2} [$,

$A_1 \cap A_2 \cap A_3 =] -1 , 1[\cap] -\frac{1}{2}, \frac{1}{2} [\cap] -\frac{1}{3}, \frac{1}{3} [=] -\frac{1}{3}, \frac{1}{3} [$ and so on.

Thus, at every step the size of the open interval is reducing and getting closure to 0,

i.e., $\bigcap_{n=1}^{\infty}(A_n) = \{0\}$ and let $F = \bigcap_{n=1}^{\infty}(A_n) = \{0\}$ then $C_L (F) = C_L (\{0\}) = \varphi$

and $I_N [\{C_L (F)\}] = I_N [\{\varphi\}] = \varphi$. But, $\{0\} \nsubseteq \varphi \implies$ $\mathbf{F \nsubseteq I_N [\{C_L (F)\}]}$.

Hence, the intersection of arbitrary collection of Pre-open set is not a Pre-open set in the topological space.

2.10 . Illustration : Let (X , J) be a topological space of a nonempty set X with the topology J then the Indiscrete topology $J = \{ \varphi, X \}$ is a Pre-open set.

Because, φ and X are an open set in the Indiscrete space and every open set is a Pre-open set in topological space.

Hence, the Indiscrete space is always a Pre-open set in the topological space.

2.11 . Illustration : Let $X \neq \varphi$ and let D be the family of all subsets of the set X then the discrete topological space (X , D) is always a Pre-open set.

Because, every subset of X is an open set in the Discrete space in topological space and we have proved earlier that every open set is a Pre-open set.

Hence, the Discrete space is always a Pre-open set in the topological space.

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