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Study on Generalization of properties on Banach Spaces.

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Abstract :

I have Studied our attention to applications are related on the study on Generalisation of properties on Banach Spaces Application to Space – theory to the properties of Banach Spaces are found in many Banaches of Barach Spaces are found in many Banaches of functional analysis in mathematical BANACHes. We apply the theory of space – theory to obtain existence and uniqueness results for properties on Banach spaces.

Introduction :

We may assume from our knowledge of modern Algebra that a particular set for two compositions of addition and Scalar multiplications is a vector space. We may also say tha it is easy to verify the postulates of a vector space for the given definitions of addition and multiplication.

In the applications of the abstract theory, it is usually show that a given differential operator. A is the infinitesimal generator of a co Semi – group in a certain concrete Banach Space of functions X. This is result provides us with the existence. and uniqueness of the initial value problem.

Banach Space :

Definition :-

A normaed linear space which is complete as a metric space is called a Banach Space. In other words a normaed linear space in which every Cauchy Sequence is convergent is called a Banach Space.

It X be a nonempty set then the mapping d: X×X→R is said to be metric if d satisties the following axioms axioms

i. $d\,(x,y) \geq o \,\forall\, x, y \in X.$

i.e. dis + *ve* realvalued function.

i.e distance between two points of X is always positive.

ii. $d(x,y) = o \Rightarrow x = y$.

i.e distance between two points of X is Zero iffthe two point coincide.

iii. $d(x,y) = d(y,x)\ \forall\ x, y \in X$

The above is known as symmetry i.e distance between x and y is the, same as the distance between y and x.

iv. $d(x,y) \leq d(x,z) + d(z,y)\ \forall\ x,y,z \in X$.

Above is Known as triangular inequality which states that the sum of two sides of a triangle is always greater than the third side only when all the three points are collinear. The metric space is denoted by (X,d).

For example : -

The space C(X) is a Banach Space : The Space C(X) is defined as the linear space of all bounded Continuous Scalar valued functions defined on X and it is a Banach Space with the norm of a function f ϵ C(X) defined as

$$\|f(x)\| = Sup.\{\ f(x) : |x \in X|\}$$

The addition and Scalar multiplication defined as.

$(f+g)x = f(x) + g(x), (\alpha f)x = \alpha[f(x)]$.

1st Step : The space C(x) is a normed linear space.

1. $\|f(x)\|$is clearly non – negative as.

   $|f(x)| > 0\ \forall x \in X$.

2. $\| f \| = 0$ $\Leftrightarrow$ Sup. $\{|f(x)| = 0\ \forall x \in X\}$

   $\Leftrightarrow |f(x)| = 0\ \forall x \in X$

   $\Leftrightarrow f(x) = 0\ \forall x \in X$.

   $\Leftrightarrow f = 0$ *ie*. Zero function.

3. $\| \alpha f \|$ = Sup. $\{|(\alpha f)(x)| : x \in X\}$

   =Sup. $\{|\alpha . f(x)| ; x \in X\}$

   =Sup. $\{|\alpha| . |f| f(x)| : x \in X\}$

$= |\alpha|$ Sup. $\{|f| : x \in X\}$

$= |\alpha| . \| f \|$

4. $\| f+g \| = \;= $ Sup. $\{|(f+g)x| : x \in X\}$

$=$Sup. $\{f(x) + g(x)| ; x \in X\}$by def.

$\leq$Sup. $\{|f(x)| + |g(x)| : x \in X\}$

$\leq$Sup. $\{|f(x)| : x \in X\}$ + Sup.$\{|g(x)| : x \in X\}$

$= \| f \| + \| g \|$

Hence.C(x) is a normal liner space.

2nd Step : Completeness of C (X).

Let $<f_m>_{m=1}^{\infty}$ be a Cauchy Sequence in C(X) therefore for $\in > 0$ there exists n_0

Such that m, p $\geq n_0$.

$\Rightarrow \| f_m - f_n \| < \in$

$\Rightarrow$Sup. $\{|(f_m - f_n)x : x \in X\} < \in$

$\Rightarrow$Sup. $\{|f_m(x) - f_n(x)| : x \in X\} < \in$

$\Rightarrow |f_m(x) - f_n(x)| < \in \; \forall x \in X.$

Above is cauchy's condition for uniform convergence of the Sequence of bounded Continuous Scalar valued founctions. Therefore sequence <fm> must be converge to a bounded continuous function f on X. Hence, C(X) is Complete and as such it is a Banach Space.

Cor. I : The space B(X) is a Banach Space

The space B(X) is defined as the linear space of all bounded Scalar valued functions defined on X and it is Banach Space with the norm of a function f ∈ B(X) defined as

$$\|f(x)\| = Sup. \{ \; f(x) : |x \in x| \}$$

Cor. II :

The liner space C [0, 1] of real valued continuous function on [0, 1] is a Banach Space with the norm of a function f ∈ C[0, 1] defined as

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$$\|f\| = max. \{I\ f(t)I : 0 \leq t \leq 1\}.$$

Theorem (1) : -

All norms on a finite dimensional linear space are equivalent.
Zeroth norm defined on a finite dimensional linear space.
Since N is finite dimensional of dimension n say there will exist a basis set consisting of n elements.

Let B = $\{e_1, e_2, \ldots\ldots e_n\}$ be basis set.

Therefore any element $x \in N$Canbe uniquely expressed as a liner combination of the elements of basis set.

i.e. $x = \sum_{i=1}^{n} \alpha i e_i$where$e_i^3$ are unique scalars.

Then the Zeroth norm on N is defined as

$$\|x\| = \max_i . \| \alpha i \|.$$

it can be easily verified that with the above definition N is a normed liner space.

Theorem (2) : -

Every finite divisional normed linear space is complete.

Let B = $\{e_1, e_2, \ldots\ldots e_n\}$ be basis for N So that

$$x = \sum_{i=1}^{n} \alpha_i\ e_i \qquad \text{———— 1.}$$

Since all norms on a finite dimensional linear space are equivalent by theorem (1) therefore it will be sufficient if we establish the completeness of N with respect to Zerothnorm defined as

$$\| x \|_0 = \max_i . \| \alpha i \| = \max_i .\ |\alpha i| \qquad \text{———— 2.}$$

Let $< yn >$ be any Cauchy Sequence in N Where

$$y_k = \sum_{i=1}^{n} \alpha i^k e_i = \alpha_i^k e_1 + \alpha_i^k e_2 + \cdots + \alpha_n^k e_n$$

Where all the n. Scalars $\alpha_i^k$are unique.

$$\therefore y_n - y_m = \sum_{i=1}^{n} (\alpha_i^n - \alpha_i^m) ei$$

$$\therefore \| y_n - y_m \|_o = \max I\ \alpha_i^n - \alpha i^m\ I \qquad 3. \text{ ————}$$

Since $< yn >$ is a canchy Sequence in N where

$\| y_n - y_m \|_o \to 0$as $m_1 n \to \infty$.

Hence from 3 we conclude that.

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$| \alpha_i^n - \alpha_i^m | \to 0$ as $m, n \to \infty$ for $i = 1, 2, \ldots\ldots n$.

But α_i^s belong to either Cor R each of which as we know is complete therefore there exist Scalers $\alpha_1, \alpha_2 \ldots\ldots \alpha_n$ such that $\alpha_i^m \to \alpha_i$ as $m \to \infty$ ($i = 1, 2 \ldots n$)

$\therefore y = \sum_{i=1}^{n} \alpha_i e_i \in N$ and $y_m \to y$.

Therefore N is Complete.

Projection on a Banach space : -

A ProjectionP on a Banach space B is an idempotent linear operator on B which is also continuous. Hence a projection P on a Banach space is a projection on a linear space B with the additional property that it is continuous.

Theorem (3) : -

Let *N* and *N'* be normed linear spaces and let *T: N*→ *N'*be any linear transformation. If N is finite dimensional then *T*is continuous.

As N is finite dimensional it must have a basis Set B = $\{e_1, e_2, e_3 \ldots\ldots e_n\}$.

$$\therefore x = \sum_{i=1}^{n} \alpha_i e_i, \; x \in N.$$

Since T is linear therefore

$$T(x) = T\left(\sum_{i=1}^{n} \alpha_i e_i\right) = \sum_{i=1}^{n} \alpha_i T(ei) \qquad 1$$

Again we know that all norms on a finite dimensional of linear space are equivalent therefore we will prove the theorem with respect to zerothnorm on N defined as

$$\|x\|_0 = \max_i |\alpha i| \qquad 2.$$

Let the norm on N' be bounded on N' be denoted by ‖‖ and T (x) ∈ N'

$$\therefore \| T(x) \| = \| \sum_{i=1}^{n} \alpha_i . T(ei) \|$$

$$\leq \sum |\alpha_i| \| T(ei) \|$$

$$\leq \|x\|_0 \sum_{i=1}^{n} \| T(ei) \| \qquad \text{[by 2]}$$

But the basis B is fixed so that $\sum_{i=0}^{n} \| T(ei) \|$is a positive constant say M. Hence from (3) we have

$$\| T(x) \| \leq M \|x\|_0.$$

Above relation shok's that T is bounded which in turn implies that T is continuous.

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