International Journal of Engineering, Science and Mathematics
Vol.8 ,Issue 1, January 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

FINITELY GENERATED GENERALIZED RIGHT ALTERNATIVE RINGS

C. Manjula, *Associate Professor in Mathematics, AMC engineering college, Bangalore-560 083
K.Jayalakshmi**Assistant Professor of Mathematics, J.N.T.UniversityAnantapur College of Engg. Anantapur.(A.P) INDIA.
Palle Prathapa Reddy***Assistant Professor in Mathematics, SJCIT engineering college, Chickballapur-562 101

ABSTRACT: *We show that weakening the hypothesis of right alternative to the three identities.*

$$(wx, y, z) + (w, x, [y, z]) = w(x, y, z) + (w, y, z)x = 0 \quad ... (1)$$

$$(x, x, x) = 0 \quad ... (2)$$

And $([x, y], y, y) = 0 ... (3)$

for all w, x, y, z*in the ring R will not lead to any new simple rings. In fact we show a semi prime finitely generated generalized right alternative ring is right alternative. We also show that if R is a prime finitelygenerated generalized right alternative ring of char.≠ 2,3.Then we show that nucleus is equal to the commutative center.*

KEYWORDS AND PHRASES:*Generalized right alternative ring, commutative center, Nucleus.*

2010 Mathematics Subject classification:17D15

INTRODUCTION: Using the standard notation,$(x, y, z) = (x, y)z - x(y, z)$, for the associative and $[x, y] = xy - yx$ for the commutator, a non associator ring which satisfies the identities.

$$\bar{A}(wx, y, z) + (w, x, [y, z]) - w(x, y, z) - (w, y, z)x = 0 \quad ... (1)$$

$$(x, x, x) = 0 \quad ... (2)$$

and$([x, y], y, y) = 0 ... (3)$

is called generalized right alternative ring and those rings that satisfy (1) and

$(x, y, z) + (y, z, x) + (z, x, y) = 0 ... (4)$

International Journal of Engineering, Science and Mathematics
Vol. 8Issue 1, January 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

are called generated non-associative ring in particular generalized right alternative ring. Straight forward verification shows that R is also generated (-1, 1) rings. On all rings that study in this paper we assume for $n = 2$ (*or*) $n = 3$, the map $x \to nx$ is one -to-one and on to. This is equivalent to char. ≠ 2, 3[8] on three conditions (1), (2), and (3) are consequences are right alternative law $(x, y, y) = 0$ and char. ≠ 2 and thus generalized right alternative rings are generalization of right alternative rings. Similar conditions have been studied by E.Kleinfeld, H.F.Smith, I.R.Hentzel and G.M.Piacentini usually through idempotent decomposition. The commutative center is defined as, $N = \{c \in R \,/\, [c, R]\} = 0$,and the nucleus N is defined as,$N=n\in R\{(n,R,R=0=R,n,R=R,R,n\}$. A number of conditions on N and also on c have been discovered which imply $N = C$. For instance, if R is simple it is so. [10].On the other hand pchelienchev has constructed a counter example in which R is strongly (-1, 1)[11].we shall here that $N = C$. Under the assumption that$R$ is prime and finitely generated.

MAIN SECTION: Throughout this paper R is a finitely generated generalized right alternative ring of char. ≠ 2,3.In such a ring the following identities are well known to hold.

$\bar{B}(w,x,y,z) = (w,x,yz) - (w,xy,z) + (w,x,yz) - w(x,y,z) - (w,x,y)z = 0$ is known as Teichmullar identity.... (5)

From equations (1) and (5), we obtain

$$\bar{C}(w,x,y,z) = (w,x,yz) - (w,xz,y) - (w,x,z)y + (w,z,y)x = 0 \ldots (6)$$

Now, applying

$\bar{A}(ab,c,y,z) + \bar{A}(a,b,y,z).C - \bar{A}(a,b,c,[y,z]) - \bar{A}(a,bc,y,z) - a.\bar{A}(b,c,y,z) - \bar{C}(a,b,[y,z],c)$ gives,

$$\bar{D}(a,b,c,y,z) = \big((a,b,c),y,z\big) - \big(a,b,(c,y,z)\big) - (a,(b,y,z),c) - \big((a,y,z),b,c\big) + (a,b,c)[y,z] - (a,b,c[y,z]) + (a,b,[y,z])c = 0 \ldots (7)$$

After equation (7), linearization of $0 = [w,([x,y],w,w)]$gives,

$$\bar{E}(x,y,z,w) = [x,([y,z],w,w)] + [w,([y,z],x,w)] + [w,([y,z],w,x)] = 0 \quad \ldots (8)$$

$$\bar{F}(y,x) = [x,(y,x,x)] + 4(y,x,x)x - 2(y,x,x^2) = 0 \ldots (9)$$

International Journal of Engineering, Science and Mathematics
Vol. 8Issue 1, January 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

and$\bar{G}(y,z,x) = -12\,(y,z,z)x - 3\{(y,x,z^2) + (y,z,x)\} + 5\{(y,z,xz) + (y,xz,z)\} + \{(y,z,zx) + (y,zx,z)\} - [x,(y,x,z)] - [z,(y,z,x)] = 0$ ... (10)

can be seen in lemmas (1) and (2),

Linearization of (2) gets,

$\bar{H}(x,y) = (x,y,y) + (y,x,y) + (y,y,x) = 0$ ... (11)

and we have, $\bar{I}(x,y) = [x^2,y] - [x,xy+yx] = 0$ ... (12)

Which is equivalent to equation (11) and

$\bar{J}(x,y,z,w) = (x,y,zw) + (x,z,wy) + (x,w,yz) - (x,yw,z) - (x,wz,y) - (x,zy,w) = 0$ ... (13)

Now, we derive, $\bar{K}(y,x) = [x,(y,x,x)] + (y,x,x)x - 2(x,x,x^2) = 0$ ... (14)

By applying $0 = -72\bar{A}(y,x,x,x)\cdot x - 18[x,\bar{A}(y,x,x,x)] + 36\,\bar{B}(y,x,x,x^2) - 3[x,\bar{C}(y,x,x,x)] + 2\,\bar{E}(x,x,y,x) + 18\,\bar{F}(yx,x) - 3[x,\bar{F}(y,x)] + 3\,\bar{G}(y,x,x^2) + 3\,\bar{I}\big(x,(y,x,x)\big) = 72\big((y,x,x),x,x\big) - 54(y,x^2,x^2) + 54(y,x,x^3) + 18\,(y,x^3,x) - 36y(x,x,x^2) - 6\big[x,[x,(y,x,x)]\big] + 6[x,([x,y],x,x)]$. By $\bar{F}$, we have $(x,x,x^2) = 0$ by equation$[x,(y,x,x)] = ([x,y],x,x)$ we have $\big[x,[x,(y,x,x)]\big] = [x,([x,y],x,x)]$. Applying all these in above, we obtain $\bar{K}(y,x) = 0$.

Next, we derive

$\bar{L}(y,x,z) = (y,x,x)(z,x,x) = 0$ ... (15)

for$\bar{L}(y,x,z) = -4(\bar{A}(y,z,x,x),x,x) - 4\,\bar{A}(y,(z,x,x),x,x) - 4\,\big((y,x,x),z,x,x\big) - y\cdot\bar{K}(z,x) - \bar{K}(y,x)\cdot z + \bar{k}(yz,x) + 3\,\bar{A}(y,z,x^2,x^2) - 2\,\bar{A}(y,z,x,x^3) - 2\,\bar{A}(y,z,x^3,x)$.

$\bar{M}(x,y,z,w) = \big([x,[y,z]],w,w\big) + \big([w,[y,z]],x,w\big) + \big([w,[y,z]],w,x\big) = 0$ ... (16)

We get $[w,([x,y],w,w)] = ([w,[x,y],w,w)$ from (1) and this is linearized to give (16).

Lemma 1: The additive subgroup *I* spanned by all associators of the form (x, y, y) where x and y range over a finitely generated right alternative ring R is a two-sided ideal of R.

Proof: We first show $IR \subseteq I$by showing $(x, y, y)z \in I$ for all $x, y, z \in R$.$0 = \bar{D}(x,y,z,y) - S(x,z,y,y) + \bar{A}(x,y,z,y) + \bar{A}(x,y,y,z) + \bar{A}(x,z,y,y) + x.\bar{G}(z,y) =$

International Journal of Engineering, Science and Mathematics
Vol. 8Issue 1, January 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

$(x, y, zy)(x, zy, y) - (x, y^2, z) - (x, z, y^2) + (xy, z, y) + (xy, y, z) + (xz, y, y) - 3(x, y, y,)z$. We have shown $3(x, y, y)z$ is a sum of associators which are in I; Since R is 2-and-3-divisible, $(x, y, y)z \in I$.This proves $I$is a right ideal, and (1) shows that I is also a left ideal.It is possible to show that if c is any element satisfying $0 = ([z, c], z, z)$ for all elements z in R (from 16), then the additive subgroup generated by all associators of the form (c, z, z) as z varies over R is a right ideal. Furthermore $([R, R], z, z)(c, z, z) = 0$. Thus $I$is a two-sided ideal of R.

Lemma 2: If R is a finitely generated generalized right alternative ring, then $((y, x, x), x, x) = 0$.

Proof: Now we write $x\big(x \circ (y, x, x)\big) = x[x(y, x, x) + (y, x, x)x]$

$$= x[(xy, x, x) + (yx, x, x)](using1)$$

$$= x[(x \circ y), x, x)]$$

$$= [x(x \circ y), x, x]$$

$$= [(x \circ y)x - x \circ (y, x), x, x](using12)$$

$$= \big((xy)x, x, x\big) + \big((yx)x, x, x\big) - (x(yx), x, x) + (x(xy), x, x) - \big((yx)x, x, x\big) + \big((xy)x, x, x\big).$$

Therefore $x\big(x \circ (y, x, x)\big) = \big((xy) \circ x, x, x\big) + \big((x, y, x), x, x\big)$.

That is, $x(x(y, x, x) + (y, x, x)x) = x \circ (xy, x, x) + \big((x, y, x), x, x\big)$.

That is, $x\big(x(y, x, x)\big) + x\big((y, x, x)x\big) = x \circ \big(x(y, x, x)\big) + \big((x, y, x), x, x\big)$.

That is, $x\big(x(y, x, x)\big) + x\big((y, x, x)x\big) = x\big(x(y, x, x)\big) + \big(x(y, x, x)\big)x + \big((x, y, x), x, x\big)$.

So $(x, (y, x, x), x) + \big((x, y, x), x, x\big) = 0$.

Then$(x, (y, x, x), x) = -\big((x, y, x), x, x\big)$

$$= \big(x, x, (x, y, x)\big) + (x, (x, y, x), x)$$

$$= \big(x, x, (x, y, x)\big) - \big((x, x, y), x, x\big)$$

$$= \big(x, x, (x, y, x)\big) + \big(x, x, (x, x, y)\big) + (x, (x, x, y), x)$$

$$= \big(x, x, (x, y, x)\big) + \big(x, x, (x, x, y)\big) - \big((x, x, x), y, x\big)$$

$$= \big(x, x, (x, y, x)\big) + \big(x, x, (x, x, y)\big)(using2)$$

That is, $(x,(y,x,x),x) = -(x,x,(y,x,x))$.

$$(x,(y,x,x),x) + (x,x,(y,x,x)) = 0.$$

Then $((y,x,x),x,x) = 0$.♦

Let us define the (-1, 1) nucleus,

K = $\{z\epsilon R \,/(R,R,z) = 0 = (x,y,z) + (y,z,x) + (z,x,y), \forall x,y\epsilon R\}$ and the hereditary (-1, 1) nucleus V of R by,

$V = \{v\epsilon K \,/\, [\ldots\ldots[[v,R],R]\ldots\ldots \subseteq K\}$Which implies$[V,R] \subseteq V$.

Let $k \in K$ and $x, y \in R$. Then,$(k,x,y) + (y,k,x) + (x,y,k) = 0$. But,$(y,k,x) = -(y,x,k) = (x,y,k)$, so that $(k,x,y) = -2(x,y,k) = 2(y,x,k) = 2(x,k,y)$. In view of this last identity we may rewrite (1) when $x = k \in K$as $(cd,y,k) + (yd,c,k) = (d,y,k)c + (d,c,k,)y$. If we assume also that,$y \in K$, then $(k,x,y) = 2(y,x,k) = 4(k,x,y)$. So that, $(k,x,y) = 0$. This proves, $(R,K,K) = (K,R,K) = 0$.

Theorem 1: A semi prime finitely generated generalized right alternative ring is a right alternative ring.

Proof: Let$\{j\epsilon R \,/j(R,x,x) = (R,x,x)j = (j,x,x) = 0\}$.It is easily seen that J is an ideal of R. From (15) we have $(y,x,x)(z,x,x) = 0$, and from lemma (1) we have $((y,x,x),x,x) = 0$. So that$(R,x,x) \subset J$. Since the annihilators of an ideal from an ideal, it is obvious that the ideal generated by (R,x,x) squares to zero. That is, $I^2 = 0$. since R is semi prime, $I = 0$. Hence R is right alternative. Similarly we can prove if R is prime ring.

♦

Theorem 2:$(R,R,K) \subseteq K$.

Proof: Let $k \in K$ and $w, x, y, z \in R$.

We substitute these elements in to (16) in three ways:

$$((w,x,k),y,z) = (w,x,(k,y,z)) + (w,(x,y,z),k) + ((w,y,z),x,k) - (w,x,k)[y,z] + (w,x,k[y,z]) - (w,x,[y,z])k. \quad \ldots (17)$$

International Journal of Engineering, Science and Mathematics
Vol. 8Issue 1, January 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

$((x,k,w),y,z)) = (x,k,(w,y,z)) + (x,(k,y,z),w) + ((x,y,z),k,w)$

$-(x,k,w)[y,z] + (x,k,w[y,z]) - (x,k,[y,z])w. \dots (18)$

$((k,w,x),y,z) = (k,w,(x,y,z)) + (k,(w,y,z),x) + ((k,y,z),w,x)$

$-(k,w,x)[yz] + (k,w,x[y,z] - (k,w,[y,z])x. \quad \dots (19)$

We add the Left hand sides of (17), (18) and (19). This equals zero since $(w,x,k) + (x,k,w) + (k,w,x) = 0$. Thus, the Right hand sides of (17), (18) and (19) must also add up to zero.

$$(w,(x,y,z),k) + \big((x,y,z),k,w\big) + \big(k,w,(x,y,z)\big) = 0 \quad \dots (20)$$

$$\big((w,y,z),x,k\big) + \big(x,k,(w,y,z)\big) + (k,(w,y,z),x) = 0 \quad \dots (21)$$

$$-(w,x,k)[y,z] - (x,k,w)[y,z] - (k,w,x)[y,z] = 0 \dots (22)$$

All follow from the definition of *K*. This leaves

$$0 = \big(w,x,(k,y,z)\big) + (x,(k,y,z),w) + \big((k,y,z),w,x\big)$$
$$+ (w,x,k[y,z]) - (w,x,[y,z])k + (w,k,w[y,z])$$
$$-(x,k,[y,z])w + (k,w,x[y,z])$$
$$-(k,w,[y,z])x. \quad \dots (23)$$

Now$(k,a,b[y,z]) - (k,a,[y,z])b = 2(a,k,b[y,z]) - 2(a,k,[y,z])b \quad \dots (24)$

Also (6) implies,

$$(a,b,k[y,z]) - (a,b,[y,z])k = -(a,k,b[y,z]) + (a,k,[y,z])b \dots (25)$$

and$(b,k,a[y,z]) - (b,k,[y,z])a = -(a,k,b[y,z]) + (a,k,[y,z])b. \dots (26)$

Since, the right hand sides of (24), (25) and (26) add up to zero, the same must be true of the left hand sides. This results in the last six terms of (23) canceling each other.

Consequently

$$0 = \big(a,b,(k,y,z)\big) + (b,(k,y,z),a) + \big((k,y,z),a,b\big). \dots (27)$$

The right nucleus is defined by,$N_r = \{n \in R \,/\, (R,R,n) = 0, \forall x \epsilon R\}$

In equation (7), let $b \epsilon N_r$,

We obtain, $(N_r, R, R) \subseteq N_r$.

So, $(k,y,z) \in N_r$

Hence, $(R, R, (k, y, z)) = 0$.Combined with (27) .This shows $(K, R, R) \subseteq K$.

♦

Corollary 1:$(R, R, V) \subseteq V$.

Proof: Since $V \subseteq K$ and $(K, R, R) \subseteq K$, we must have $(V, R, R) \subseteq K$.

Let, k $\in K$,$x, y, z \in R$. Use of (1) results in,

$$(xy, z, k) + (x, y, [y, z]) = x(y, z, k) + (x, z, k)y, \quad \ldots (28)$$

$$(yx, z, k) + (y, x, [z, k]) = (y, z, k)x + y(x, z, k). \ldots (29)$$

Subtract (29) from (28). Thus

$([x, y], z, k) + (x, y, [z, k]) - (y, x, [z, k]) = [x, (y, z, k)] - [y, (x, z, k)]$,or

$$([x, y], z, k) + (x, y, [z, k]) - (y, x, [z, k]) = [x, (y, z, k)] + [y, (z, x, k)]. \quad \ldots (30)$$

Permute$x$, y, $z$cyclicly in (30).Then

$$([y, z], x, k) + (y, z, [x, k]) - (z, y, [x, k]) = [y, (z, x, k)] + [z, (x, y, k)]. \ldots (31)$$

Permuting$y$, x,$z$cyclicly in (31) results in

$$([z, x], y, k) + (z, x, [y, k]) - (x, z, [y, k]) = [z, (x, y, k)] + [x, (y, z, k)]. \ldots (32)$$

Adding (30) and (32) and subtracting (31) results in

$$2[x, (y, z, k)] = ([x, y], z, k) - ([y, z], x, k) + ([z, x], y, k) + (x, y, [z, k]) - (y, x, [z, k]) - (y, z, [x, k]) + (z, y, [x, k]) + (z, x, [y, k]) - (x, z, [y, k]). \quad \ldots (33)$$

If we assume $k \in V$ in equation (33), then in fact (32) gives

$$[R, (R, R, V)] \subseteq (R, R, V) \subseteq k. \quad \ldots (34)$$

Now using (34), we have $[R, [R, (R, R, V)]] \subseteq [R, (R, R, V)] \subseteq (R, R, V) \subseteq k$.

Further commutations with R collapse in the same way.

We conclude that $(R, R, V) \subseteq V$. ♦

Definition 1.$Let S = \sum(R, R, V) + (R, R, V)R$.

Definition 2. Let $T = \{t \epsilon R / t(R, R, V) = 0 = (R, R, V)t = (R, t, V)\}$

Lemma 3:S and T are ideals of R, and $S \neq 0$ implies$T = 0$ when R is prime.

Proof: Let $t \epsilon T, x, y, z, \epsilon R$we obtain $v \in V$.

Then use of (1) results in,

$(tx, y, v) + (t, x, [y, v]) = t(x, y, z) + (t, y, v)x$.

Now, since that, $[y, v] \epsilon V$.

It follows that, $(tx, y, v) = 0$

Similarly, $(xt, y, v) = 0$

Using the Corollary, we obtain, $tx \cdot (y, z, v) = t \cdot x(y, z, v)$

And using result (1), we get$t \cdot x(y, z, v) = t \cdot (x, y, z, v) + t \cdot (x, y, [z, v]) - t \cdot (y, z, v)y = 0$.

i.e. $TR \subseteq T$. Also, we have$xt \cdot (y, z, v) = x \cdot t(y, z, v)$

Again, using result (1), we obtain $x \cdot t(y, z, v) = (ty, z, v) + (t, y, [z, v]) - x(t, z, v)y$.

$$0 = x \cdot t(y, z, v)$$
$$= xt \cdot (y, z, v)$$

i. e.$R\ T \subseteq T$. Thus, T is an ideal of$R$.

Next we show that, S is an ideal of R.

Now from corollary (1)as$(R, R, V) \subseteq V$. We have $(V, R, R) \subseteq V$. Thus from (1) we have $R(R, R, V) \subseteq S$

Also, $\big((R, R, V), R, R\big) \subseteq (V, R, R) \subseteq S$.which shows that S is an ideal of R.

The definitions of $T$and S implies that, $TS = 0$.

Since, R is prime it follows that, $S \neq 0$ implies $T = 0$. ♦

Corollary 2: If $(R, R, V) \neq 0$ then $[V, V] = 0$ when R is prime.

Proof: We have $(R, R, V) \subseteq S$, thus $(R, R, V) \neq 0$ implies $S \neq 0$, whence$T = 0$ by the previous lemma.Letv, v^*, v_1, v_2, v_3 ϵ V, and $x, y \epsilon R$. Then

$$[xv, v^*] = x[v, v^*] + [x, v^*]v + 2\,(x, v, v^*) + (v^*, x, v)$$
$$= x[v, v^*] + [x, v^*]v_1$$

Using a standard identity and $(R, R, V) = 0$.Thus using (1),

$$(vx, y, v^*) + (v, x, [y, v^*]) = v(x, y, v^*) + (v, y, v^*)x.$$

Thus $(vx, y, v^*) = v(x, y, v^*)$. Then

$$[v_1, v_2](x, y, v_3) = ([v_1, v_2]x, y, v_3)$$
$$= ([v_1\, x, v_2], y, v_3) - (v_1[x, v_2], y, v_3)$$
$$= -(v_1[x, v_2], y, v_3) = 0,$$

Using (1) and $(R, V, V) = 0$.A similar argument shows $(x, y, v_3)[v_1, v_2] = 0$.

Consequently $[V, V] \subseteq T = 0$. This concludes the proof of the corollary. ♦

Lemma 4: If R is prime and finitely generated by n elements, then$(R, R, V)^{n+1} = 0$.

Proof: From the Corollary to Lemma 1 it follows that [*V*, *V*] = 0. From the Corollary to Theorem 1

It follows that $(V, R, R) \subseteq V$. Consequently for$w, x, y, z \epsilon R, v_1, v_2 \epsilon V$the associators (w, x, v_1)

and(y, z, v_2)commute and any three associators of this form associate. Then

$$(w, x, v_1)(y, z, v_2) = \big((w, x, v_1)y, z, v_2\big) + \big((w, x, v_1), y, [z, v_2]\big) - ((w, x, v_1), z, v_2)y$$
$$= ((w, x, v_1)\, y, z, v_2).$$

Now using (6) and $(v_1, a, b) = 2(b, a, v_1)$ we have

$$(w, x, v_1)y + (y, x, v_1)w = (w, yx, v_1) + (y, wx, v_1),$$

So that$((w, x, v_1)y + (y, x, v_1)w, z, v_2) = \big((w, yx, v_1), z, v_2\big) = 0,$

Whence $\big((w, x, v_1)y, z, v_2\big) = -\big((y, x, v_1)w, z, v_2\big) = -(y, x, v_1)(w, z, v_2)$

Thus

$$(w, x, v_1)(y, z, v_2) = -(y, x, v_1)(w, z, v_2). \quad \dots (35)$$

Combining this with $(y, z, v_2) = -(z, y, v_2)$, we get

$$(\pi(w), \pi(x), v_1).(\pi(y), \pi(z), v_2) = sgn(\pi)(w, x, v_1)(y, z, v_2) \dots (36)$$

If the elements of$R$ are generators throughout then clearly $(R, R, V)^{n+1} = 0$, for if any two associators have the same element d then we can bring those associators next to each other and (28) with char.$\neq 2$ makes the product zero. More generally suppose the elements from$R$ are words. Consider an element in $\prod_{I=1}^{n+1}(R, R, V)$, and write it as $q\,(x, y, v)$ where

$q \in \prod_{i=1}^{n}(R, R, V)$.

Suppose$x = w_1 w_2$, the product of two words w_1 and w_2.

Then implies$(w_1 w_2, y, v)$ = - q $(yw_2, w_1, v) + q \cdot (w_2, w_1, v)$ y + q $\cdot(w_2, y, v)$ w_1. ... (37)

Since $-q(yw_2, w_1, v)$ = q(w_1, yw_2, v), what was x is now replaced by w_1, a word of shorter length.

Note that q$\cdot$ $(w_2, w_1, v)y = q(w_2, w_1, v) \cdot yandq \cdot (w_2, y, v)w_1 = q(w_2, y, v) \cdot w_1$.Also the components of these associators are words of shorter lengths as well. If we now write $w_1$as aProduct of two words of shorter length using (37), we eventually work our way down to a generator, where x was. One can do that with all components, using one component of associator as the garbagecollector, so to speak. In any case we have proved the lemma.

♦

Theorem 3: If R is finitely generated and prime then $(R, R, V) = 0$.

Proof: Assume $(R, R, V) \neq 0$. Then Lemma 2 implies $(R, R, V)^{N+1} = 0$. Consequently $(R, R, V)^n \subseteq T$. But $T = 0$ by Lemma 1. Thus $(R, R, V)^n$ = 0.Repeated use of this argument gives,$(R, R, V)^{n-1}$ = 0, $(R, R, V)^{n-2}$ = 0, until at last $(R, R, V) = 0$.This contradicts $(R, R, V) \neq 0$, Hence, $(R, R, V) = 0$. This completes the proof the Theorem.♦

Theorem 4: Let R be a finitely generated, prime right alternative ring of characteristic $\neq 2,3$.Then the nucleus equals to commutator center.

Proof: Let $x, y \epsilon R$ and $c \epsilon C$.

Then Jacobi identity, $[[c, x], y] + [x, y], c] + [[y, c], x] =$

$$0 = 2(c, x, y) + 2(y, c, x) + 2(x, y, c) = 2((c, x, y) + 2(y, c, x) + 2(x, y, c)$$
$$= (c, x, y) + 2(y, c, x) + 2(x, y, c).$$

Also, we have the semi Jacobi identity,

$[x, y, z] - x[y, z] - [x, z]y - (x, y, z) - (z, x, y) + (x, z, y) = 0$.

Put, $z = c$ in the above, we have, $0 = -(x, y, c) - (c, x, y) + (x, c, y) = -2(x, y, c) - (c, x, y)$.

i.e. $2(x, y, c) = 0$.

Thus, $(x, y, c) = 0$.Hence, $c \epsilon K$ the (-1, 1) nucleus.

Now, $c \epsilon K$ and $[c, R] = 0$implies $c \epsilon V$, thus $C \subseteq V$.

Now, using theorem (2), $(R, R, C) = 0$.

Since, $[R, C] = 0$ to begin with we have proved $C \subseteq N$.

Now, to show that, $N \subseteq C$, we define,

$$(x \circ y) = xy + yx$$

Then, $(x \circ y) \circ z - x \circ (y \circ z) = (x, y, z) + (x, z, y) + (y, z, x) - (y, z, x)$

$$-(z, x, y) - (z, y, x) + (y, [x, z])$$

Implies$(y, [x, z]) = 0$ since, C is a commutative.

By taking, $z = n \epsilon N$

We have, $[[R, R], N] = 0$

i.e.$[R, N] = 0$.Thus, $N \subseteq C$. But, then that $N = C$. This completes the proof. ♦

REFERENCES

[1] A.A.Albert,Almost alternative algebras Portugal Math.8(1949), 23-36.

[2] A. Thedy, Right alternative rings, J. algebra 37 (1975), 1-43.

[3] ErwinKleinfeld and Harry F.Smith, on centers and nuclei in prime right alternative rings, Comm.Algebra 22(3)(1994), 829-855.

[4] E.Kleinfeld, generalization of alternative rings, I.J.Algebra, 18(1971), 304-325.

[5] ……...Generalization of alternative rings, II. J. Algebra, 18(1971), 326-339.

[6] ……... A Generalization of (-1,1) rings, Pacific J.Math., 53(1974), 195-202.

[7] H.F.Smith, Prime generalized alternative rings with non-trivial idempotent, Proc.Amer.math.SocS. 39(1973), 242-246.

[8] I.R. Hentzel, Characteristic in non-associative ring.

[9]I.R. Hentzel and G.M.Piacentini, Generalization of right alternative rings, Trans.Amer.Math.Soc.207 (1975), 143-161.

[10]R.E.Roomledi, on simple right alternative ring (Russian), Tartu Riikl.$\bar{U}I$.Toimetised No. 700 (1985), 60-70.

[11] S.v.Pchelincev, Prime algebras and absolute divisors of zero(Russian), lzv.Akad.Nauk SSSRSer.Mat. 50(1986),79-100.