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AN INCREMENTAL ALGORITHM TO COMPUTE SHANNON ENTROPY FOR DYNAMIC INFORMATION SYSTEM

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ABSTRACT

KEYWORDS:

Rough set;
Shannon entropy;
Incremental approach;
Information system;
Attribute reduction

Since the classical attribute reduction of rough sets is difficult to obtain, it is popular to search attribute reduction based on information entropies. In this paper, an incremental method to update Shannon conditional entropy for dynamic information systems with varying objects is studied. Then an incremental attribute reduction algorithm for decision table with varying object set is proposed. A numerical experiment is given to show the efficiency of the new method.

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1. INTRODUCTION

As a powerful soft compute tool, rough set has received much attention since its appearance [3, 6, 7, 11]. As we all know, the essence of knowledge is classification, but the criterion for classification is the attribute set. Thus, how to search the attribute reduction is a hot topic in rough set [1, 4, 14, 18]. Because computing the classical reduction is an NP hard problem, researchers often turns to alternatives: to search a attribute set which can maintain information entropy. As a result, many heuristic algorithms have been developed for attribute reduction based on information entropy [8, 10, 12, 13, 20].

In real world, the information system continues changing with time goes by. More commonly, the objects set keep on increasing. As a result, incremental algorithm to search attribute reduction for dynamic information system is a hot topic [2, 9, 16, 17]. Particularly, Wang et al. [16] discussed how to update entropy for dynamic information system when changing data values and they gave some useful recursion formulas. However, the object set and attribute set must remain unchanged in their work, which limit its wide application. Furthermore, their updating mechanisms can deal with situation in which attribute values are changed one by one. Their ideas are reasonable but their work needs further study. Follow their way, we study an incremental approach to update Shannon conditional entropy for dynamic information systems with varying object set.

So as to facilitate our discussion, the remainder of this paper is arranged as follows. Some basic concepts about rough set are recalled in Section 2. In Section 3, we discuss an incremental approach to compute Shannon conditional entropy for dynamic information systems. An incremental attribute reduction algorithm for dynamic information systems is proposed in Section 4. Section 5 give a numerical experiment to testify the new algorithm. Finally, a brief conclusion and future work directions are given in Section 6.

2. PRELIMINARIES

In this section, we recall some basic concepts of rough sets and information entropy.

Definition 2.1[5] An information system (or decision table) is defined as a pair $\langle U, A\rangle$ where U is a non-empty finite set of objects, $A = C \cup D$ is a non-empty finite set of attributes, C denotes the set of condition attributes and D denotes the set of decision attributes, $C \cap D = \emptyset$. Each attribute $a \in A$ is associated with a set V_a of its value, called the domain of a.

The concept of information system provides a convenient framework to represent the objects in the universe. Actually, every $B \subseteq A$ can determine an equivalence relation R_B as

$$\forall x, y, R_B(x, y) \Leftrightarrow \forall a \in B: a(x) = a(y),$$

where $a(x)$ denotes the attribute value of x respect to a. Every R_B can partition U into some equivalence classes given by $U/R_B = \{[x]_B | x \in U\}$, or just U/B for short, where $[x]_B = \{y \in U | (x, y) \in R_B\}$ denotes the equivalence class including x with respect to B.

Shannon entropy is a famous measure in information theory. Wang et al. [15] further introduced Shannon conditional entropy to find a reduct of an information system.

Definition 2.2[16] Let $S = (U, C \cup D)$ be an information system, $B \subseteq C$, $U/B = \{X_1, X_2, \cdots X_m\}$ and $U/D = \{Y_1, Y_2, \cdots Y_n\}$. Shannon's conditional entropy of B relative to D is defined as

$$H_U(D|B) = -\sum_{i=1}^{m} \left(\frac{|X_i|}{|Y_j|}\right) \sum_{j=1}^{n} \frac{|X_i \cap Y_j|}{|X_i|} \log \frac{|X_i \cap Y_j|}{|X_i|}.$$

The entropy reduction was proposed by Skowron in 1993. Then, Slezak [19] introduced Shannon conditional entropy to get reducts in classical rough set model.

Definition 2.3[16] Let $S = (U, C \cup D)$ be information system and $B \subseteq C$. Then, B is a relative reduct if it satisfies:

(1) $H(D|B) = H(D|C)$;

(2) $\forall a \in B, H(D|B - \{a\}) \neq H(D|B)$

The first condition indicates that B keeps the same entropy as C, and the second condition indicates that each attribute in B is individually necessary.

3 AN INCREMENTAL APPROACH TO UPDATING SHANNON CONDITIONAL ENTROPY FOR DYNAMIC INFORMATION SYSTEMS

Let $S=(U, C\cup D)$ be an information system and $B\subseteq C$. The Shannon's conditional entropy of D with respect to B is $H_U(D|B)$. In this section, we shall establish a mathematical fundamental to compute Shannon's conditional entropy for dynamic data sets.

Lemma 3.1 Suppose only one object x get out of the system S, and $x\in X_{p_1}, x\in Y_{q_1}$.Then we get a new information system S' and $U'=U-\{x\}$, $U'/B=\{X_1,X_2,\cdots X'_{p_1},\cdots X_m\}$, $U'/D=\{Y_1,Y_2,\cdots Y'_{q_1},\cdots Y_n\}$, where $X'_{p_1}=X_{p_1}-\{x\}$, $Y'_{q_1}=Y_{q_1}-\{x\}$. New Shannon conditional entropy $H_{U'}(D|B)$ can be given as

$$\frac{(|U|-1)}{|U|}\cdot H_{U'}(D|B)$$

$$=H_U(D|B)-\sum_{j=1,j\neq q1}^{n}\frac{X_{p_1}\cap Y_j|}{|U|}\log\frac{|X_{p_1}|}{|X_{p_1}|-1}-\frac{|X_{p_1}\cap Y_{q1}|}{|U|}\log\left(\frac{|X_{p_1}|(|X_{p_1}\cap Y_{q1}|-1)}{X_{p_1}\cap Y_{q1}(|X_{p_1}|-1)}\right)$$

$$-\frac{1}{|U|}\log\frac{|X_{p_1}|-1}{|X_{p_1}\cap Y_{q1}|-1}$$

$$\approx H_U(D|B)-\frac{1}{|U|}\log\frac{|X_{p_1}|-1}{|X_{p_1}\cap Y_{q1}|-1}.$$

Proof. By Definition 2.2, we have

$$H_{U'}(D|B)$$

$$=-\sum_{i=1,i\neq p_1}^{m}\frac{|X_i|}{|U'|}\sum_{j=1,j\neq q_1}^{n}\frac{|X_i\cap Y_j|}{|X_i|}\log\frac{|X_i\cap Y_j|}{|X_i}-\sum_{i=1,i\neq p_1}^{m}\frac{|X_i|}{|U'|}\frac{|X_i\cap Y'_{q1}|}{|X_i|}\log\frac{|X_i\cap Y'_{q1}|}{|X_i|}$$

$$-\sum_{j=1,j\neq q_1}^{n}\frac{|X'_{p1}|}{|U'|}\frac{|X'_{p1}\cap Y_j|}{|X'_{p1}|}\log\frac{|X'_{p1}\cap Y_j|}{|X'_{p1}|}-\frac{|X'_{p1}|}{|U'|}\frac{|X'_{p1}\cap Y'_{q1}|}{|X'_{p1}|}\log\frac{|X'_{p1}\cap Y'_{q1}|}{|X'_{p1}|}.$$

Multiply this equation by $\frac{(|U|-1)}{|U|}$ and use $X'_{p_1}=X_{p_1}-\{x\}$, $Y'_{q_1}=Y_{q_1}-\{x\}$,

$$\frac{(|U|-1)}{|U|}\cdot H_{U-\{x\}}(D|B)$$
$$=-\sum_{i=1}^{m}\frac{|X_i|}{|U|}\sum_{j=1}^{n}\frac{|X_i\cap Y_j|}{|X_i|}\log\frac{|X_i\cap Y_j|}{|X_i}+\sum_{i=1,i\neq p_1}^{m}\frac{|X_i|}{|U|}\frac{|X_i\cap Y_{q1}|}{|X_i|}\log\frac{|X_i\cap Y_{q1}|}{|X_i|}$$
$$+\sum_{j=1,j\neq q_1}^{n}\frac{|X_{p1}|}{|U|}\frac{|X_{p1}\cap Y_j|}{|X_{p1}|}\log\frac{|X_{p1}\cap Y_j|}{|X_{p1}|}+\frac{|X_{p1}|}{|U|}\frac{|X_{p1}\cap Y_{q1}|}{|X_{p1}|}\log\frac{|X_{p1}\cap Y_{q1}|}{|X_{p1}|}$$
$$-\sum_{i=1,i\neq p_1}^{m}\frac{|X_i|}{|U|}\frac{|X_i\cap Y_{q1}|}{|X_i|}\log\frac{|X_i\cap Y_{q1}|}{|X_i|}-\sum_{j=1,j\neq q_1}^{n}\frac{|X_{p1}|-1}{|U|}\frac{|X_{p1}\cap Y_j|}{|X_{p1}|-1}\log\frac{|X_{p1}\cap Y_j|}{|X_{p1}|-1}$$
$$-\frac{|X_{p1}|-1}{|U|}\frac{|X_{p1}\cap Y_{q1}|-1}{|X_{p1}|-1}\log\frac{|X_{p1}\cap Y_{q1}|-1}{|X_{p1}|-1}$$
$$=H_U(D|B)-\sum_{j=1,j\neq q1}^{n}\frac{|X_{p_1}\cap Y_j|}{|U|}\log\frac{|X_{p_1}|}{|X_{p_1}|-1}-\frac{|X_{p_1}\cap Y_{q1}|}{|U|}\log(\frac{|X_{p_1}|(|X_{p_1}\cap Y_{q1}|-1)}{|X_{p_1}\cap Y_{q1}|(|X_{p_1}|-1)})$$
$$-\frac{1}{|U|}\log\frac{|X_{p_1}|-1}{|X_{p_1}\cap Y_{q1}|-1}.$$

In large-scale information system [16], $|X_{p_1}|, |Y_{q1}|, |X_{p_1}\cap Y_{q1}|$ are relatively large, so we can get $\log\frac{|X_{p_1}|}{|X_{p_1}|-1}\approx 0$, $\log\frac{|Y_{q1}|}{|Y_{q1}|-1}\approx 0$, $\log\frac{|X_{p1}\cap Y_{q1}|}{|X_{p1}\cap Y_{q1}|-1}\approx 0$, thus the above formula can be simplified to $\frac{(|U|-1)}{|U|}\cdot H_{U'}(D|B)\approx H_U(D|B)-\frac{1}{|U|}\log\frac{|X_{p_1}|-1}{|X_{p_1}\cap Y_{q1}|-1}$. When an object is removed or added, we can update Shannon conditional entropy easily.

Lemma 3.2 Suppose only one object x enter the system S, and $x\in X_{p_2}, x\in Y_{q_2}$. Then we get a new information system S' and $U'=U\cup\{x\}$, $U'/B=\{X_1,X_2,\cdots X'_{p_2},\cdots X_m\}$, $U'/D=\{Y_1,Y_2,\cdots Y'_{q_2},\cdots Y_n\}$, where $X'_{p_2}=X_{p_2}\cup\{x\}$, $Y'_{q_2}=Y_{q_2}\cup\{x\}$. $H_{U'}(D|B)$ can be computed as

$$\frac{(|U|+1)}{|U|}\cdot H_{U'}(D|B)$$
$$=H_U(D|B)+\sum_{j=1,j\neq q2}^{n}\frac{X_{p_2}\cap Y_j|}{|U|}\log\frac{|X_{p_2}|+1}{|X_{p_2}|}+\frac{|X_{p_2}\cap Y_{q2}|}{|U|}\log(\frac{X_{p_2}\cap Y_{q2}(|X_{p_2}|+1)}{|X_{p_2}|(|X_{p_2}\cap Y_{q2}|+1)})$$
$$+\frac{1}{|U|}\log\frac{|X_{p_2}|+1}{|X_{p_2}\cap Y_{q2}|+1}$$
$$\approx H_U(D|B)+\frac{1}{|U|}\log\frac{|X_{p_2}|+1}{|X_{p_2}\cap Y_{q2}|+1}.$$

Proof. Since

$$H_{U'}(D|B)$$
$$=-\sum_{i=1,i\neq p_2}^{m}\frac{|X_i|}{|U'|}\sum_{j=1,j\neq q_2}^{n}\frac{|X_i\cap Y_j|}{|X_i|}\log\frac{|X_i\cap Y_j|}{|X_i}|-\sum_{i=1,i\neq p_2}^{m}\frac{|X_i|}{|U'|}\frac{|X_i\cap Y'_{q2}|}{|X_i|}\log\frac{|X_i\cap Y'_{q2}|}{|X_i|}$$
$$-\sum_{j=1,j\neq q_2}^{n}\frac{|X'_{p2}|}{|U'|}\frac{|X'_{p2}\cap Y_j|}{|X'_{p2}|}\log\frac{|X'_{p2}\cap Y_j|}{|X'_{p2}|}-\frac{|X'_{p2}|}{|U'|}\frac{|X'_{p2}\cap Y'_{q2}|}{|X'_{p2}|}\log\frac{|X'_{p2}\cap Y'_{q2}|}{|X'_{p2}|},$$

multiplying this equation by $\frac{(|U|+1)}{|U|}$ and replacing X'_{p_2}, Y'_{q_2} with $X_{p_2} \cup \{x\}$, $Y_{q_2} \cup \{x\}$ respectively.

$$\frac{(|U|+1)}{|U|} \cdot H_{U\cup\{x\}}(D|B)$$

$$= -\sum_{i=1}^{m} \frac{|X_i|}{|U|} \sum_{j=1}^{n} \frac{|X_i \cap Y_j|}{|X_i|} \log \frac{|X_i \cap Y_j|}{|X_i} + \sum_{i=1,i\neq p_2}^{m} \frac{|X_i|}{|U|} \frac{|X_i \cap Y_{q2}|}{|X_i|} \log \frac{|X_i \cap Y_{q2}|}{|X_i|}$$

$$+ \sum_{j=1,j\neq q_2}^{n} \frac{|X_{p2}|}{|U|} \frac{|X_{p2} \cap Y_j|}{|X_{p2}|} \log \frac{|X_{p2} \cap Y_j|}{|X_{p2}|} + \frac{|X_{p2}|}{|U|} \frac{|X_{p2} \cap Y_{q2}|}{|X_{p2}|} \log \frac{|X_{p2} \cap Y_{q2}|}{|X_{p2}|}$$

$$- \sum_{i=1,i\neq p_2}^{m} \frac{|X_i|}{|U|} \frac{|X_i \cap Y_{q2}|}{|X_i|} \log \frac{|X_i \cap Y_{q2}|}{|X_i|} - \sum_{j=1,j\neq q_2}^{n} \frac{|X_{p2}|+1}{|U|} \frac{|X_{p2} \cap Y_j|}{|X_{p2}|+1} \log \frac{|X_{p2} \cap Y_j|}{|X_{p2}+1|}$$

$$- \frac{|X_{p2}|+1}{|U|} \frac{|X_{p2} \cap Y_{q2}|+1}{|X_{p2}|+1} \log \frac{|X_{p2} \cap Y_{q2}|+1}{|X_{p2}|+1}$$

$$= H_U(D|B) + \sum_{j=1,j\neq q2}^{n} \frac{X_{p_2} \cap Y_j|}{|U|} \log \frac{|X_{p_2}|+1}{|X_{p_2}|} + \frac{|X_{p_2} \cap Y_{q2}|}{|U|} \log(\frac{X_{p_2} \cap Y_{q2}(|X_{p_2}|+1)}{|X_{p_2}|(|X_{p_2} \cap Y_{q2}|+1)})$$

$$+ \frac{1}{|U|} \log \frac{|X_{p_2}|+1}{|X_{p_2} \cap Y_{q2}|+1}.$$

In large-scale information system [16], because $|X_{p_2}|, |Y_{q2}|, |X_{p_2} \cap Y_{q2}|$ are relatively large, we can consider $\log \frac{|X_{p_2}|}{|X_{p_2}|-1} \approx 0$, $\log \frac{|Y_{q_2}|}{|Y_{q_2}|-1} \approx 0$, $\log \frac{|X_{p2} \cap Y_{q_2}|}{|X_{p2} \cap Y_{q_2}|-1} \approx 0$, then the above formula can be simplified to $\frac{(|U|+1)}{|U|} \cdot H_{U\cup\{x\}}(D|B) \approx H_U(D|B) + \frac{1}{|U|} \log \frac{|X_{p_2}|+1}{|X_{p_2} \cap Y_{q2}|+1}$.

As our inference, we can obtain Wang's Theorem as follows. This implies that our approach can be widely used.

Theorem 3.3[16] If one and only one object $x \in U$ is changed to x', then $x' \in X'_{p_2}$, $x \in Y'_{q_2}$. The new Shannon conditional entropy becomes

$$H_{U-\{x\}\cup\{x'\}}(D|B)$$

$$= H_U(D|B) + \sum_{j=1,j\neq q2}^{n} \frac{X_{p_2} \cap Y_j|}{|U|} \log \frac{|X_{p_2}|+1}{|X_{p_2}|} + \frac{|X_{p_2} \cap Y_{q2}|}{|U|} \log(\frac{X_{p_2} \cap Y_{q2}(|X_{p_2}|+1)}{|X_{p_2}|(|X_{p_2} \cap Y_{q2}|+1)})$$

$$+ \frac{1}{|U|} \log \frac{|X_{p_2}|+1}{|X_{p_2} \cap Y_{q2}|+1} - \sum_{j=1,j\neq q1}^{n} \frac{X_{p_1} \cap Y_j|}{|U|} \log \frac{|X_{p_1}|}{|X_{p_1}|-1}$$

$$- \frac{|X_{p_1} \cap Y_{q1}|}{|U|} \log(\frac{|X_{p_1}|(|X_{p_1} \cap Y_{q1}|-1)}{X_{p_1} \cap Y_{q1}(|X_{p_1}|-1)}) - \frac{1}{|U|} \log \frac{|X_{p_1}|-1}{|X_{p_1} \cap Y_{q1}|-1}.$$

Proof. For the change of x to x', we pull out in two stages as follows. Firstly, we remove x from S, then the Shannon's conditional entropy of $H_{U-\{x\}}(D|B)$ can be computed By Lemma 3.1, that is

$$\frac{(|U|-1)}{|U|}\cdot H_{U'}(D|B)$$
$$= H_U(D|B) - \sum_{j=1,j\neq q1}^{n} \frac{X_{p_1}\cap Y_j|}{|U|}\log\frac{|X_{p_1}|}{|X_{p_1}|-1} - \frac{|X_{p_1}\cap Y_{q1}|}{|U|}\log(\frac{|X_{p_1}|(|X_{p_1}\cap Y_{q1}|-1)}{X_{p_1}\cap Y_{q1}(|X_{p_1}|-1)})$$
$$-\frac{1}{|U|}\log\frac{|X_{p_1}|-1}{|X_{p_1}\cap Y_{q1}|-1}.$$

Secondly, we add x' to the object set $U-\{x\}$, and we get the new Shannon's conditional entropy $H_{U'}(D|B) = H_{U-\{x\}\cup\{x'\}}(D|B)$. Thus from Lemma 3.2,

$$\frac{(|U|)}{|U|-1}\cdot H_{U-\{x\}\cup\{x'\}}(D|B)$$
$$= H_{U-\{x\}}(D|B) + \sum_{j=1,j\neq q2}^{n} \frac{X_{p_2}\cap Y_j|}{|U|-1}\log\frac{|X_{p_2}|+1}{|X_{p_2}|} + \frac{|X_{p_2}\cap Y_{q2}|}{|U|-1}\log(\frac{X_{p_2}\cap Y_{q2}(|X_{p_2}|+1)}{|X_{p_2}|(|X_{p_2}\cap Y_{q2}|+1)})$$
$$+\frac{1}{|U|-1}\log\frac{|X_{p_2}|+1}{|X_{p_2}\cap Y_{q2}|+1},$$

Solve these two equations and eliminate $H_{U-\{x\}}(D|B)$, we can complete the proof.

When M objects get out of the system, we can delete these objects one by one, and obtain the following theorem.

Lemma 3.4 If M objects get out of the system, M_{ij} represents that there are M_i objects which remove from the conditional class X_i and M_{ij} objects of them come from decision class Y_j. Then the new Shannon's conditional entropy which we denote by $H_{U-M}(D|B)$ becomes

$$\frac{(|U|-M)}{|U|}\cdot H_{U-M}(D|B) \approx H_U(D|B) - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{M_{ij}}\frac{1}{|U|}\log\frac{|X_i|-k}{|X_i\cap Y_j|-k}).$$

Proof. We can remove these M objects from S one by one. When there are M_{11} objects get out, we can use the approximate formula in Lemma 3.1 for M_{11} times. Thus,

$$\frac{(|U|-M_{11})}{|U|}\cdot H_{U-M_{11}}(D|B) \approx H_U(D|B) - \sum_{k=1}^{M_{11}}\frac{1}{|U|}\log\frac{|X_1|-k}{|X_1\cap Y_1|-k}.$$

Next, we remove M_{12} objects from this new system. We make use of the approximate formula in Lemma 3.1 for M_{12} times.

That is

$$\frac{(|U|-M_{11}-M_{12})}{(|U|-M_{11})}\cdot H_{U-M_{11}-M_{12}}(D|B) \approx H_{U-M_{11}}(D|B) - \sum_{k=1}^{M_{12}}\frac{1}{|U|-M_{11}}\log\frac{|X_1|-k}{|X_1\cap Y_2|-k}.$$

Repeating this operation, we can easily complete this proof by induction axioms.

When N objects enter the system, we can add these objects one by one,and give the following theorem.

Lemma 3.5 If N objects enter the system, N_{ij} represents that there are N_i objects enter the conditional class X_i and N_{ij} objects of them enter decision class Y_j. Then the new Shannon's conditional entropy which we denote by $H_{U+N}(D|B)$ becomes $\frac{(|U|+N)}{|U|}\cdot H_{U+N}(D|B) \approx H_U(D|B) + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{N_{ij}}\frac{1}{|U|}log\frac{|X_i|+k}{|X_i\cap Y_j|+k}$.

Proof. We can add these N objects into S one by one. When there are N_{11} objects get in, we can use the approximate formula in Lemma 3.2 for N_{11} times. Thus,

$$\frac{(|U|+N_{11})}{|U|}\cdot H_{U+N_{11}}(D|B)=H_U(D|B)+\sum_{k=1}^{N_{11}}\frac{1}{|U|}\log\frac{|X_1|+k}{|X_1\cap Y_1|+k}.$$

Similarly, we add N_{12} objects into this new system, then

$$\frac{(|U|+N_{11}+N_{12})}{(|U|+N_{11})}\cdot H_{U+N_{11}+N_{12}}(D|B)\approx H_{U+N_{11}}(D|B)+\sum_{k=1}^{N_{12}}\frac{1}{|U|+N_{11}}\log\frac{|X_1|+k}{|X_1\cap Y_2|+k}$$

In general, we add N_{ij} objects into the latest system. Thus,

$$\frac{|U|+N_{11}+\cdots+N_{ij}}{|U|+N_{11}+\cdots+N_{i,j-1}}\cdot H_{U+N_{11}+\cdots+N_{ij}}(D|B)=H_{U+N_{11}+\cdots+N_{i,j-1})}(D|B)$$
$$+\sum_{k=1}^{N_{ij}}\frac{1}{|U|+N_{11}+\cdots+N_{i,j-1}}\log\frac{|X_i|+k}{|X_i\cap Y_j|+k}.$$

By induction axioms, we can easily complete this proof.

We obtain the conclusion below by combining Lemma 3.4 and Lemma 3.5.

Theorem 3.6 *If N objects enter the system, and M objects get out of the system at the same time. The new Shannon's conditional entropy which we denote by $H_{U+N-M}(D|B)$ becomes*

$$\frac{(|U|+N-M)}{|U|}\cdot H_{U+N-M}(D|B)$$
$$\approx H_U(D|B)-\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{M_{ij}}\frac{1}{|U|}\log\frac{|X_i|-k}{|X_i\cap Y_j|-k})+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{N_{ij}}\frac{1}{|U|}\log\frac{|X_i|+k}{|X_i\cap Y_j|+k}.$$

Proof. We first add these N objects into S one by one. By Lemma 3.5 , we get

$$\frac{(|U|+N)}{|U|}\cdot H_{U+N}(D|B)\approx H_U(D|B)+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{N_{ij}}\frac{1}{|U|}\log\frac{|X_i|+k}{|X_i\cap Y_j|+k}.$$

Then we remove M objects from the system above, by Lemma 3.4,

$$\frac{(U+N-M)}{|U|+N}\cdot H_{U+N-M}(D|B)\approx H_{U+N}(D|B)-\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{M_{ij}}\frac{1}{|U|+N}\log\frac{|X_i|-k}{|X_i\cap Y_j|-k}).$$

Combine these two equations and eliminate $H_{U+N}(D|B)$, we complete our proof.

4 AN INCREMENTAL ATTRIBUTE REDUCTION ALGORITHM FOR DECISION TABLE WITH DYNAMICALLY VARYING OBJECT SET

Based on recursion formula 3.6, here, we will propose an incremental attribute reduction algorithm for dynamically varying object set. Firstly, we need to run the classic algorithm CAR[16] and store the result U/B, U/D, $H(D|B)$. Then, we add or delete some objects and perform our incremental algorithm as follows.

Algorithm: An incremental attribute reduction algorithm for decision table with dynamically varying object set (IAR)

Input: A decision table $S = (U, C \cup D)$, N new objects enter S and M objects escape from S.

Output: Attribute reduction RED_{U+N-M} of new decision table $S' = (U + N - M, C \cup D)$.

Step 1: Run classical attribute reduction algorithm and store U/B, U/D, $H(D|B)$ for $B \subseteq C$.

Step 2: For each $x \in N$ or $x \in M$, check $x \in X_i$ and $x \in Y_i$, where $X_i \in U/B$, $Y_j \in U/D$. Then we get N_{ij} and M_{ij}, $i = 1,2,\cdots,|U/B|, j = 1,2,\cdots,|U/D|$.

Step 3: $reduct \leftarrow \phi$, $i \leftarrow 1$.

Step 4: According to Theorem 3.6, we compute $H_{U+N-M}(D|reduct)$.

Step 5: If $H_{U+N-M}(D|reduct) = H_{U+N-M}(D|C)$, then turn to step 7; else turn to step 6.

Step 6: While $(H_{U+N-M}(D|reduct) \neq H_{U+N-M}(D|C))$ do

{

$i \leftarrow i + 1$;

$reduct \leftarrow reduct \cup c_i$, where c_i is the i-th important attribute in C.

}

Step 7: $RED_{U+N-M} \leftarrow reduct$, return RED_{U+N-M} and end.

Form above algorithm, we can clearly find that, we do not need to compute U/B, U/D, $H(D|B)$ repeatedly. In other words, the message in CAR is used in IAR. Because it is extremely time-consuming to compute $H(D|B)$, the efficiency of IAR is improved.

5 EXPERIMENTAL ANALYSIS

Here, we give a numerical example to verify the efficiency of IAR. The data set named Chess (King-Rook vs. King-Pawn).txt is downloaded from UCI repository of machine learning databases. The experiment is carried out on a personal computer with window 7 and Intel(R) Core(TM) i5-5200 CPU, 2.20GHz, 12 GB memory, with Mathematica 8.0 software.

First, we take out the first $|U| = 2000$ objects from the data set. Then, we perform the classical reduction algorithm CAR. Next, we add 100 new objects step by step and perform IAR comparing with CAR. The consumed time by CAR and IAR can be clearly compared by the following figure.

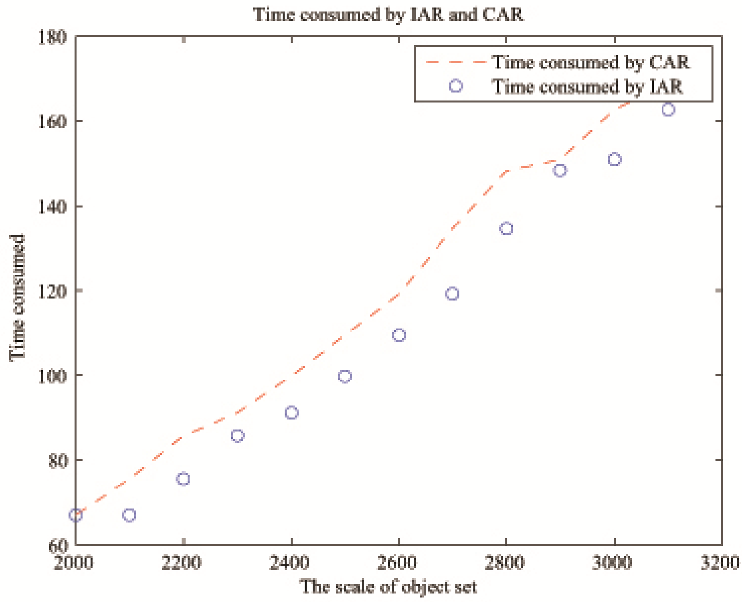

Figure 1: Time consumed by IAR and CAR.

Obviously, IAR works much better than CAR here. The reason maybe that, the existing information acquired by CAR such as U/B, U/D, $H(D|B)$, $B \subseteq C$ is used by IAR. When new objects are added, we only need to update $H_{U+N}(D|B)$ by Theorem3.6. As a result, time consumed by IAR is less than CAR.

6 CONCLUSIONS AND FUTURE WORK

It is a hot topic to search attribute reduction by heuristic algorithms based on entropies. As a result, how to update entropy incrementally is the core task. In this paper, we propose an incremental method to update Shannon conditional entropy for dynamic information systems with varying objects. A numerical example has shown the efficiency of the newly built method. In addition, the variation of data values can be treated as our special case. However, the new method is applicable when attribute sets do not change. In the future, we will research how to update information entropy for dynamic information system with varying attribute sets.

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