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# SINGLE SERVER FEEDBACK MARKOVIAN QUEUEING SYSTEM WITH IMPATIENT CUSTOMERS AND TWO TYPES OF VACATIONS

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**Abstract**

We consider an $M/M/1$ queue with two types of vacations and Bernoulli feedback simultaneously. During both vacations, the customers become impatient. That is whenever a customer arrives at the system it activates an 'impatience timer'. If the customer's service has not been completed before the customer's timer expire, the customer leaves the queue and never return. The stationary probability distribution of the system studied by probability generating function technique and some performance measures are also derived.

***Keywords:***

***Markovian Queue,***

***Vacations,***

***Feedbacks,***

***Impatient Customers.***

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## 1. Introduction

Consider a system operating as an $M/M/1$ queue. In recent years, the study of vacation queues has a great effect on the queueing theory. Queueing system with server vacations have been extensively studied from $1970's$ onwards by various researchers. This system is applied in various fields such as computer system, communication networks etc..The appication of vacation model is available in the survey paper of Doshi[8], Wu and Takagi[23] and Tian and Zhang[24].

Waiting for service is usually an unpleasent experience and represents the loss of valuable resources, which translates into psychoogical as well as economic cost of waiting. The concept of waiting server with server vacation was first introduced by Boxma $etal$. [3]. There is now growing interest in the analysis of queueing systems with impatience customers. Laxmi and Jyothsna have discussed lot about the impatient customers in [25]. Queueing system with impatience customers reflects in many real life queueing system, particularly when dealing with human behaviour. Customers impatience has been dealt with in the queueing literature mainly in the context of customers abandoning the queue due to either a long wait alread experienced or a long wait anticipated upon arrival.

Altman and Yechiali [1] have discussed briefly about the impatience customers with single and multiple working vacations. They have also discussed the $M/M/\infty$ queueing model with impatient customers and vacations in [2]. Gang et al.[11] have extended [2] with impatience customers and working vacations. They have derived the probability geneating function of number of customers in the system and calculated values of key performances. Goswami[5] and Ammar[19] have discussed about two differentiated vacations. In [18], Padmavathi et al. have discussed impatience customers with vacation and a waiting server in their paper. In [26], Yue et al. analysed Markovian queueing systems with impatient customers and a variant of multiple vacation policy.

One more feature which has been widely studied in queueing systems is feedback. The concept of Bernoulli feedback is widely applied in computer time sharing and telecommunications systems. Tackas [21] was the first to study feedback queueing model. Further studies on the queue length, the total sojourn time and waiting time are provided by Disney et al. [6], [7]. Fontana and Berzova [10] have extended some results obtained for the $M/G/1$ model with Bernoulli feedback to a more general feedback with priorities.

Disney et al.[7] have given an overview of literature concerning Bernoulli feedback studies. Queues with feedback exhibit interesting and some what unexpected behaviour. However, such problems are widely studied. One may refer to Falin [9], Kumar et al. [15], Ke and Chang [13], Kumar et al. [16] and Li Tao et al. [12].

The remaining part of the paper is organised as follows: In section 2, we describe the model description. We develop the probabaility generating functions of the steady state probabilities and solving the differential equations, we get a closed form expressions of the mean system size when it is in different states in section 3, 4 and 5, respectively. We also obtain expression for other performance measures in section 6.

## 2. *Model description and analysis*

We consider an $M/M/1$ queueing system with impatient customers and two vacations. Customers arrive according to a Poisson process at rate $\lambda$. The service is provided by a single server, who serves the customer on FCFS basis. The service time follows an exponential distribution with a service rate $\mu$. After each service, the customer may, independent of the remaining stochastic behaviour of the system, return to the queue with probability $\delta(0 < \delta < 1)$ or depart from the system with probability $1 - \delta$.

When the server finishes service and finds the system empty, he does not leave for vacation and he stays ideal for a random period and then he leaves the system with rate $\alpha$. If the server finds a customer at a vacation completion, the server returns to serve a customer immediately. Otherwise the server will take another vacation with rate $\gamma_1$. After the completion of second vacation, if there is no customer, the server returns to first vacation with rate $\gamma_2$. If he found customer arrives after completion of second vacation, immediately he start serving the customer.

During the vacations, the customers become impatient. That is whenever a customer arrives at the system it activates an 'imapatience timer' T, which is exponentially distributded with parameters $\zeta_1$, $\zeta_2$ respectively. If the customer's service has not been completed before the customer's timer expire, the customer leaves the queue and never return. We can find, at any time $t$, the system can be completely described by the following two random variables: $L(t)$ and $S(t)$, where $L(t)$ represents the number of customers in the system at time $t$, $S(t)$ speaks the state of server at time $t$, which is defined as follows: $S(t) = 0$ denotes that the server is busy or idle at time $t$, for $k = 1,2$, $S(t) = k$ denotes that the server is in $k$th vacation. Then $\{(S(t), L(t)), t \geq 0\}$ is a continuous time Markov chain that has state space

$$\Omega = \{(s, j), s = 0,1,2, j \geq 0\}$$

We define the following steady-state probabilities:

$P_{sj}$ = The probability that there are $j$ customers in the system when the service station is in the state $s$

We establish the steady-state equations for our model in the next section.

***2.1Steady state analysis***

The steady-state equations for $P_{sj}$, $s = 0,1,2$ and $j \geq 0$ are given by

$$(\lambda + \alpha)P_{00} = \mu\delta P_{01} \quad (1)$$

$$(\lambda + \mu\delta)P_{0n} = \lambda P_{0n-1} + \mu\delta P_{0n+1} + \gamma_1 P_{1n} + \gamma_2 P_{2n} \quad (2)$$

$$(\lambda + \gamma_1)P_{10} = \alpha P_{00} + \zeta_1 P_{11} + \gamma_2 P_{20} \quad (3)$$

$$(\lambda + \gamma_1 + n\zeta_1)P_{1n} = \lambda P_{1n-1} + (n+1)\zeta_1 P_{1n+1} \quad (4)$$

$$(\lambda + \gamma_2)P_{20} = \zeta_2 P_{21} + \gamma_1 P_{10} \quad (5)$$

$$(\lambda + \gamma_2 + n\zeta_2)P_{2n} = \lambda P_{2n-1} + (n+1)\zeta_2 P_{2n+1} \quad (6)$$

Let $P_s(z) = \sum_{j=0}^{\infty} P_{sj} z^j, |z| \leq 1$ and $s = 0,1,2$ be the generating function for the probability $P_s$, then by (1) - (6) and some habitual algebraic manipulations, we get

$$[\lambda(1-z) + \mu\delta(1-\tfrac{1}{z})]P_0(z) = \gamma_1 P_1 z + \gamma_2 P_2(z) - [\alpha - \mu\delta(1-\tfrac{1}{z})]P_{00}$$
$$-\gamma_1 P_{10} - \gamma_2 P_{20} \quad (7)$$

$$(z-1)\zeta_1 P_1'(z) + [\lambda(1-z) + \gamma_1]P_1(z) = \alpha P_{00} + \gamma_2 P_{20} \quad (8)$$

$$(z-1)\zeta_2 P_2'(z) + [\lambda(1-z) + \gamma_2]P_2(z) = \gamma_1 P_{10} \quad (9)$$

***2.2Solution of differential equations***

Equation (8) can be written as

$$P_1'(z) - \left[\frac{\lambda}{\zeta_1} + \frac{\gamma_1}{\zeta_1(1-z)}\right]P_1(z) = -\left[\frac{\alpha P_{00} + \gamma_2 P_{20}}{\zeta_1(1-z)}\right] \quad (10)$$

In order to solve the differential equation(10), we multiply both sides by

$$e^{-\frac{\lambda}{\zeta_1}z}(1-z)^{\frac{\gamma_1}{\zeta_1}}$$

Then, we get

$$\frac{d}{dz}\left[e^{-\frac{\lambda}{\zeta_1}z}(1-z)^{\frac{\gamma_1}{\zeta_1}}P_1(z)\right] = -\left[\frac{\alpha P_{00}+\gamma_2 P_{20}}{\zeta_1}\right]e^{-\frac{\lambda}{\zeta_1}z}(1-z)^{\frac{\gamma_1}{\zeta_1}-1} \quad (11)$$

Integrating from 0 to $z$, we have

$$P_1(z) = \frac{e^{\frac{\lambda}{\zeta_1}z}\left\{P_{10}-\left[\frac{\alpha P_{00}+\gamma_2 P_{20}}{\zeta_1}\right]\int_0^z e^{-\frac{\lambda}{\zeta_1}s}(1-s)^{\frac{\gamma_1}{\zeta_1}-1}ds\right\}}{(1-z)^{\frac{\gamma_1}{\zeta_1}}} \quad (12)$$

Since $z=1$ is the root of the denominator of the R.H.S of equation(12), we have that $z=1$ must be the root of numerator of the R.H.S. Therefore we have

$$P_{10} = \left[\frac{\alpha P_{00}+\gamma_2 P_{20}}{\zeta_1}\right]C_1 \quad (13)$$

$$where C_1 = \int_0^1 e^{-\frac{\lambda}{\zeta_1}s}(1-s)^{\frac{\gamma_1}{\zeta_1}-1}ds$$

Substituting equation (13) in equation (12), we obtain

$$P_1(z) = \frac{e^{\frac{\lambda}{\zeta_1}z}}{(1-z)^{\frac{\gamma_1}{\zeta_1}}}\frac{\alpha\zeta_2}{\zeta_1\zeta_2-\gamma_1\gamma_2 C_1 C_2}C_1\left[1-\frac{1}{C_1}\int_0^z e^{-\frac{\lambda}{\zeta_1}s}(1-s)^{\frac{\gamma_1}{\zeta_1}-1}ds\right]P_{00} \quad (14)$$

Equation (9) can be written as

$$P_2'(z) - \left[\frac{\lambda}{\zeta_2}+\frac{\gamma_2}{\zeta_2(1-z)}\right]P_2(z) = -\left[\frac{\gamma_1 P_{10}}{\zeta_2(1-z)}\right] \quad (15)$$

In order to solve the differential equation (10), we multiply both sides by

$$e^{-\frac{\lambda}{\zeta_2}z}(1-z)^{\frac{\gamma_2}{\zeta_2}}$$

In a similar manner used for solving equation (8), we get

$$P_2(z) = \frac{e^{\frac{\lambda}{\zeta_2}z}\left\{P_{20}-\left[\frac{\gamma_1 P_{10}}{\zeta_2}\right]\int_0^z e^{-\frac{\lambda}{\zeta_2}s}(1-s)^{\frac{\gamma_2}{\zeta_2}-1}ds\right\}}{(1-z)^{\frac{\gamma_2}{\zeta_2}}} \quad (16)$$

Since $z=1$ is the root of the denominator of the R.H.S of equation (16), we have that $z=1$ must be the root of numerator of the R.H.S. Therefore we have

$$P_{20} = \left[\frac{\gamma_1 P_{10}}{\zeta_2}\right]C_2 \quad (17)$$

$$where C_2 = \int_0^1 e^{-\frac{\lambda}{\zeta_2}s}(1-s)^{\frac{\gamma_2}{\zeta_2}-1}ds$$

substituting equation (17) in equation (16), we get

$$P_2(z) = \frac{e^{\frac{\lambda}{\zeta_2}z}}{(1-z)^{\frac{\gamma_2}{\zeta_2}}} \frac{\alpha\gamma_1}{\zeta_1\zeta_2-\gamma_1\gamma_2C_1C_2} C_1C_2\left[1 - \frac{1}{C_2}\int_0^z e^{-\frac{\lambda}{\zeta_2}s}(1-s)^{\frac{\gamma_2}{\zeta_2}-1}ds\right] P_{00} \tag{18}$$

Equations(14) and (18) in terms of $P_{00}$. Therefore once $P_{00}$ is calculated, the $P_1(z)$ and $P_2(z)$ are determined. In the next section, we derive the probabilty $P_{00}$ and the mean system sizes when the server is in different states.

### *2.3Mean system sizes*

Let $L_1$ be the system size when the server is in the state 1. Then $E(L_1)$ is the mean system size when the server is in the state 1, which is defined by

$$E(L_1) = \sum_{n=1}^{\infty} nP_{1n}$$

From equation (8), using L'Hospital rule, we obtain

$$P_1'(1) = \lim_{z\to 1} \frac{[\lambda(1-z)+\gamma_1]P_1(z)-(\alpha P_{00}+\gamma_2P_{20})}{(1-z)\zeta_1}$$

$$= \frac{\gamma_1P_1'(1)-\lambda P_1(1)}{-\zeta_1} \tag{19}$$

Thus, we get

$$E(L_1) = \frac{\lambda}{\zeta_1+\gamma_1}P_1(1) \tag{20}$$

Similarly, let $L_2$ be the system size when the server is in the state 2. Then, $E(L_2)$ is the mean system size when the server is in the state 2, which is defined by $E(L_2) = \sum_{n=1}^{\infty} nP_{2n}$ From equation (9), using L'Hospital rule, we get

$$P_2'(1) = \lim_{z\to 1} \frac{[\lambda(1-z)+\gamma_2]P_2(z)-(\gamma_1P_{10})}{(1-z)\zeta_2}$$

$$= \frac{\gamma_2P_2'(1)-\lambda P_2(1)}{-\zeta_2} \tag{21}$$

Thus, we get

$$E(L_2) = \frac{\lambda}{\zeta_2+\gamma_2}P_2(1) \tag{22}$$

Furthermore, the mean system size when the server is on vacation, denoted by $E(L_v)$, is obtained as follows

$$E(L_v) = E(L_1) + E(L_2)$$

$$= \frac{P_{00}}{\zeta_1\zeta_2-\gamma_1\gamma_2C_1C_2}\left[\frac{\lambda}{\zeta_1+\gamma_1}\frac{e^{\frac{\lambda}{\zeta_1}}\alpha\zeta_1\zeta_2}{\gamma_1} + \frac{\lambda}{\zeta_2+\gamma_2}\frac{e^{\frac{\lambda}{\zeta_2}}\alpha\gamma_1\zeta_2}{\gamma_2}C_1\right] \tag{23}$$

Equation (7) can be written as

$$P_0(z) = \frac{\gamma_1P_1(z)+\gamma_2P_2(z)-\gamma_1P_{10}-\gamma_2P_{20}-[\alpha-\mu\delta(1-\frac{1}{z})]P_{00}}{\lambda(1-z)+\mu\delta(1-\frac{1}{z})} \tag{24}$$

Applying L'Hospital rule, we get

$$P_0(1) = \frac{\gamma_1 E(L_1) + \gamma_2 E(L_2) + \mu\delta P_{00}}{\mu\delta - \lambda} \tag{25}$$

Substituting equations (20) and (22), we get

$$P_0(z) = \frac{P_{00}}{\mu\delta-\lambda}\left[\frac{\lambda}{\zeta_1+\gamma_1}\frac{e^{\frac{\lambda}{\zeta_1}}\alpha\zeta_1\zeta_2}{\gamma_1} + \frac{\lambda}{\zeta_2+\gamma_2}\frac{e^{\frac{\lambda}{\zeta_2}}\alpha\gamma_1\zeta_2}{\gamma_2}C_1 + \mu\delta\right] \tag{26}$$

Using normalization condition

$$P_0(1) + P_1(1) + P_2(1) = 1$$

we arrive at

$$P_{00} = [\zeta_1\zeta_2 - \gamma_1\gamma_2 C_1 C_2]\left\{e^{\frac{\lambda}{\zeta_1}}\alpha\zeta_1\zeta_2\left[\frac{1}{\gamma_1} + \frac{1}{\mu\delta-\lambda}\frac{\lambda}{\zeta_1+\gamma_1}\right]\right.$$
$$\left.+e^{\frac{\lambda}{\zeta_2}}\alpha\gamma_1\zeta_2 C_1\left[\frac{1}{\gamma_2} + \frac{1}{\mu\delta-\lambda}\frac{\lambda}{\zeta_2+\gamma_2}\right] + \mu\delta(\zeta_1\zeta_2 - \gamma_1\gamma_2 C_1 C_2)\right\}^{-1} \tag{27}$$

Substituting equation (27) in equation (23), we get

$$E(L_v) = \left[\frac{\lambda}{\zeta_1+\gamma_1}\frac{e^{\frac{\lambda}{\zeta_1}}\alpha\zeta_1\zeta_2}{\gamma_1} + \frac{\lambda}{\zeta_2+\gamma_2}\frac{e^{\frac{\lambda}{\zeta_2}}\alpha\gamma_1\zeta_2}{\gamma_2}C_1\right]\left\{e^{\frac{\lambda}{\zeta_1}}\alpha\zeta_1\zeta_2\left[\frac{1}{\gamma_1} + \frac{1}{\mu\delta-\lambda}\frac{\lambda}{\zeta_1+\gamma_1}\right]\right.$$
$$\left.+e^{\frac{\lambda}{\zeta_2}}\alpha\gamma_1\zeta_2 C_1\left[\frac{1}{\gamma_2} + \frac{1}{\mu\delta-\lambda}\frac{\lambda}{\zeta_2+\gamma_2}\right] + \mu\delta(\zeta_1\zeta_2 - \gamma_1\gamma_2 C_1 C_2)\right\}^{-1} \tag{28}$$

Now we derive $E(L_0)$ from the equation (24), using L'Hospital rule

$$E(L_0) = \frac{\gamma_1 P_1''(1) + \gamma_2 P_2''(1) - 2\mu\delta P_{00}}{2(\mu\delta-\lambda)} + \frac{\mu\delta}{(\mu\delta-\lambda)^2}[\gamma_1 P_1'(1) + \gamma_2 P_2'(1) + \mu\delta P_{00}]$$
$$-\frac{3\mu\delta}{(\mu\delta-\lambda)^2}[\gamma_1 P_1(1) + \gamma_2 P_2(1) - \gamma_1 P_{10} - \gamma_2 P_{20} - \alpha P_{00}] \tag{29}$$

where $P_1''(1)$ and $P_2''(1)$ is obtained by differentiating twice $P_1(z)$ and $P_2(z)$ at $z = 1$. Differentiating twice the equations (14) and (18) respectively with $z = 1$, we obtain

$$P_1''(1) = \frac{\lambda P_1'(1)}{\zeta_1} \tag{30}$$

$$and P_2''(1) = \frac{\lambda P_2'(1)}{\zeta_2} \tag{31}$$

Substituting equation (30) and (31) in (29), we get

$$E(L_0) = \frac{1}{\mu\delta-\lambda}\left\{\gamma_1 P_1(1)\left[\left(\frac{\lambda}{2\zeta_1} + \frac{\mu\delta}{\mu\delta-\lambda}\right)\left(\frac{\lambda}{\zeta_1+\gamma_1}\right) - \frac{3\mu\delta}{\mu\delta-\lambda}\right]\right.$$
$$\left.+\gamma_2 P_2(1)\left[\left(\frac{\lambda}{2\zeta_2} + \frac{\mu\delta}{\mu\delta-\lambda}\right)\left(\frac{\lambda}{\zeta_2+\gamma_2}\right) - \frac{3\mu\delta}{\mu\delta-\lambda}\right]\right\}$$
$$+\frac{1}{(\mu\delta-\lambda)^2}[3\mu\delta\gamma_1 P_{10} + 3\mu\delta\gamma_2 P_{20} + 3\mu\delta\alpha P_{00} + \mu\delta P_{00}] \tag{32}$$

Let L be the number of customers in the system. Then, the mean system size

$$E(L) = E(L_v) + E(L_0) \tag{33}$$

can be calculated from the equations (32) and (28).

$$E(L) = \frac{P_{00}}{\zeta_1\zeta_2 - \gamma_1\gamma_2 C_1 C_2}\left[\frac{\lambda}{\zeta_1+\gamma_1}\frac{e^{\frac{\lambda}{\zeta_1}}\alpha\zeta_1\zeta_2}{\gamma_1} + \frac{\lambda}{\zeta_2+\gamma_2}\frac{e^{\frac{\lambda}{\zeta_2}}\alpha\gamma_1\zeta_2}{\gamma_2}C_1\right]$$
$$+\frac{1}{\mu\delta-\lambda}\left\{\gamma_1 P_1(1)\left[\left(\frac{\lambda}{2\zeta_1}+\frac{\mu\delta}{\mu\delta-\lambda}\right)\left(\frac{\lambda}{\zeta_1+\gamma_1}\right)-\frac{3\mu\delta}{\mu\delta-\lambda}\right]\right.$$
$$\left.+\gamma_2 P_2(1)\left[\left(\frac{\lambda}{2\zeta_2}+\frac{\mu\delta}{\mu\delta-\lambda}\right)\left(\frac{\lambda}{\zeta_2+\gamma_2}\right)-\frac{3\mu\delta}{\mu\delta-\lambda}\right]\right\}$$
$$+\frac{1}{(\mu\delta-\lambda)^2}\left[3\mu\delta\gamma_1 P_{10} + 3\mu\delta\gamma_2 P_{20} + 3\mu\delta\alpha P_{00} + \mu\delta P_{00}\right]$$

***Some performance measures***

In this section, we derive some other performance measures.

**1. Probabilty that the server is on vacation**

The probability that the server is on vacation is given by

$$P_v = \sum_{n=0}^{\infty} P_{1n} + \sum_{n=0}^{\infty} P_{2n}$$
$$= P_1(1) + P_2(1) \qquad (34)$$

Substituting the values of $P_1(1)$ and $P_2(1)$ in equation (31)

$$P_v = \frac{P_{00}}{\zeta_1\zeta_2-\gamma_1\gamma_2 C_1 C_2}\left[\frac{e^{\frac{\lambda}{\zeta_1}}\alpha\zeta_1\zeta_2}{\gamma_1} + \frac{e^{\frac{\lambda}{\zeta_2}}\alpha\gamma_1\zeta_2 C_1}{\gamma_2}\right] \qquad (35)$$

**2. Probabilty that the server is busy**

The probability that the server is busy is given by

$$P_b = \sum_{n=1}^{\infty} P_{0n} - P_{00} \qquad (36)$$

Substituting the values of $P_0(1)$, we get

$$P_b = P_{00}\left[\frac{1}{\mu\delta-\lambda}\left(\frac{\lambda}{\zeta_1+\gamma_1}\frac{e^{\frac{\lambda}{\zeta_1}}\alpha\zeta_1\zeta_2}{\zeta_1\zeta_2-\gamma_1\gamma_2 C_1 C_2} + \frac{\lambda}{\zeta_2+\gamma_2}\frac{e^{\frac{\lambda}{\zeta_2}}\alpha\gamma_1\zeta_2 C_1}{\zeta_1\zeta_2-\gamma_1\gamma_2 C_1 C_2} + \mu\delta\right) - 1\right] \qquad (37)$$

Where

$$P_{00} = [\zeta_1\zeta_2 - \gamma_1\gamma_2 C_1 C_2]\left\{e^{\frac{\lambda}{\zeta_1}}\alpha\zeta_1\zeta_2\left[\frac{1}{\gamma_1}+\frac{1}{\mu\delta-\lambda}\frac{\lambda}{\zeta_1+\gamma_1}\right]\right.$$
$$\left.+e^{\frac{\lambda}{\zeta_2}}\alpha\gamma_1\zeta_2 C_1\left[\frac{1}{\gamma_2}+\frac{1}{\mu\delta-\lambda}\frac{\lambda}{\zeta_2+\gamma_2}\right] + \mu\delta(\zeta_1\zeta_2 - \gamma_1\gamma_2 C_1 C_2)\right\}^{-1}$$

**3. Proportion of customers served**

Clearly, the expected number of customers served per unit of time is $\mu\delta P_b$, implying that the proportion of customers served is given by

$$P_s = \frac{\mu\delta}{\lambda} P_b \qquad (38)$$

where $P_b$ is given by the equation(38).

**4. Average rate of abandonment due to impatience**

When the system is in state $(1,n)$, $n \geq 1$, the rate of abandonment of a customer due to impatience is $n\zeta_1$ and in state $(2,n)$, $n \geq 1$, the rate of abandonment of a customer due to impatience is $n\zeta_2$. Thus the average rate of abandonment due to impatience is given by

$$R_a = \sum_{n=1}^{\infty} n\zeta_1 P_{1n} - P_{10} + \sum_{n=1}^{\infty} n\zeta_2 P_{2n} - P_{20} \qquad (39)$$

**Conclusion**

A single server queueing model which has Poisson arrivals and exponentially distributed service times with two types of vacations, customers' impatience and feedback is analyzed in steady state regime. The explicit solution for the system size probabilities is obtained. Additionally, the mean number of customers in the system in steady state as the performance measures.

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