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# **Group Theory and Its Application** Fundamental Concepts of Modern Algebra

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## ABSTRACT

The purpose of this work is to provide a concise yet detailed **Group Theory and Its Application** of fundamental concepts in modern algebra. **Modern algebra**, also called **abstract algebra**, branch of mathematics concerned with the general algebraic structure of various sets (such as real numbers, complex numbers, matrices, and vector spaces), rather than rules and procedures for manipulating their individual elements. During the second half of the 19th century, various important mathematical advances led to the study of sets in which any two elements can be added or multiplied together to give a third element of the same set. The elements of the sets concerned could be numbers, functions, or some other objects. As the techniques involved were similar, it seemed reasonable to consider the sets, rather than their elements, to be the objects of primary concern. A definitive treatise, *Modern Algebra*, was written in 1930 by the Dutch mathematician Bartel van der Waerden, and the subject has had a deep effect on almost every branch of mathematics. The target audience for this research work is first-year graduate students in mathematics, though the first two chapters are probably accessible to well-prepared undergraduates. This Research work a broad range of topics in modern algebra and includes on groups, rings, modules, algebraic extension fields, and finite fields. This Work begins with an overview which provides a road map for the reader showing what material will be covered. At the end of each chapter we collect exercises which review and reinforce the material in the corresponding sections. These exercises range from straightforward applications of the material to problems designed to challenge the reader. We also include a list of "Questions for Further Study" which pose problems suitable for master's degree research projects.

**Keywords:** Modern Algebra, Abstract Algebra, Structure, Applications, Group.

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## Introduction

## Basic algebraic structures

### Fields

In itself a set is not very useful, being little more than a well-defined collection of mathematical objects. However, when a set has one or more operations (such as addition and multiplication) defined for its elements, it becomes very useful. If the operations satisfy familiar arithmetic rules (such as associativity, commutativity, and distributivity) the set will have a particularly "rich" algebraic structure. Sets with the richest algebraic structure are known as fields. Familiar examples of fields are the rational numbers (fractions $a/b$ where $a$ and $b$ are positive or negative whole numbers), the real numbers (rational and irrational numbers), and the complex numbers (numbers of the form $a + bi$ where $a$ and $b$ are real numbers and $i^2 = -1$). Each of these is important enough to warrant its own special symbol: $\mathbb{Q}$ for the rationals, $\mathbb{R}$ for the reals, and $\mathbb{C}$ for the complex numbers. The term *field* in its algebraic sense is quite different from its use in other contexts, such as vector fields in mathematics or magnetic fields in physics. Other languages avoid this conflict in terminology; for example, a field in the algebraic sense is called a *corps* in French and a *Körper* in German, both words meaning "body."

In addition to the fields mentioned above, which all have infinitely many elements, there exist fields having only a finite number of elements (always some power of a prime number), and these are of great importance, particularly for discrete mathematics. In fact, finite fields motivated the early development of abstract algebra. The simplest finite field has only two elements, 0 and 1, where 1 + 1 = 0. This field has applications to coding theory and data communication.

### Structural axioms

The basic rules, or axioms, for addition and multiplication are shown in the table, and a set that satisfies all 10 of these rules is called a field. A set satisfying only axioms 1–7 is called a ring, and if it also satisfies axiom 9 it is called a ring with unity. A ring satisfying the commutative law of multiplication (axiom 8) is known as a commutative ring. When axioms 1–9 hold and there are no proper divisors of zero (i.e., whenever $ab = 0$ either $a = 0$ or $b = 0$), a set is called an integral domain. For example, the set of integers {…, −2, −1, 0, 1, 2, …} is a commutative ring with unity, but it is not a field, because axiom 10 fails. When only axiom 8 fails, a set is known as a division ring or skew field.

**Prime factorization**

Some other fundamental concepts of modern algebra also had their origin in 19th-century work on number theory, particularly in connection with attempts to generalize the theorem of (unique) prime factorization beyond the natural numbers. This theorem asserted that every natural number could be written as a product of its prime factors in a unique way, except perhaps for order (e.g., 24 = 2∙2∙2∙3). This property of the natural numbers was known, at least implicitly, since the time of Euclid. In the 19th century, mathematicians sought to extend some version of this theorem to the complex numbers.

One should not be surprised, then, to find the name of Gauss in this context. In his classical investigations on arithmetic Gauss was led to the factorization properties of numbers of the type $a + ib$ ($a$ and $b$ integers and $i$ = Square root of$\sqrt{-1}$), sometimes called Gaussian integers. In doing so, Gauss not only used complex numbers to solve a problem involving ordinary integers, a fact remarkable in itself, but he also opened the way to the detailed investigation of special subdomains of the complex numbers. In 1832 Gauss proved that the Gaussian integers satisfied a generalized version of the factorization theorem where the prime factors had to be especially defined in this domain. In the 1840s the German mathematician Ernst Eduard Kummer extended these results to other, even more general domains of complex numbers, such as numbers of the form $a + \theta b$, where $\theta^2 = n$ for $n$ a fixed integer, or numbers of the form $a + \rho b$, where $\rho^n = 1$, $\rho \neq 1$, and $n > 2$. Although Kummer did prove interesting results, it finally turned out that the prime factorization theorem was not valid in such general domains. The following example illustrates the problem.

## Research Methodology

**Group theory,** in modern algebra, the research methdology which are systems consisting of a set of elements and a binary operation that can be applied to two elements of the set, which together satisfy certain axioms. These require that the group be closed under the operation (the combination of any two elements produces another element of the group), that it obey the associative law, that it contain an identity element (which, combined with any other element, leaves the latter unchanged), and that each element have an inverse (which combines with an element to produce the identity element). If the group also satisfies the commutative law, it is called a commutative, or abelian, group. The set of integers under addition, where the identity element is 0 and the inverse is the negative of a positive number or vice versa, is an abelian group. Groups are vital to modern algebra; their basic structure can be found in many mathematical phenomena. Groups can be found in geometry, representing phenomena such as symmetry and certain types of transformations. Group theory has applications in physics, chemistry, and computer science, and even puzzles like Rubik's Cube can be represented using group theory. On the lighter side, there are applications of group theory to puzzles, such as the 15-puzzle and Rubik's Cube. Group theory provides the conceptual framework for solving such puzzles. To be fair, you can learn an algorithm for solving Rubik's cube without knowing group theory (consider this 7-year old cubist), just as you can learn how to drive a car without knowing automotive mechanics. Of course, if you want to understand how a car works then you need to know what is really going on under the hood. Group theory (symmetric groups, conjugations, commutators, and semi-direct products) is what you find under the hood of Rubik's cube Periodic materials, like crystals, have translational symmetry. The translation operation $T_{\vec{r}}$ leaves the lattice invariant.

## Methods & Material

In this paper We have trying to find the best result and current research work on the basics of objectives. This current work and preliminary work done on these guide lines. My current is

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Mathematical problems in **algebra** have been resolved with group theory. In the Renaissance, mathematicians found analogues of the quadratic formula for roots of general polynomials of degree 3 and 4. Like the quadratic formula, the cubic and quartic formulas express the roots of all polynomials of degree 3 and 4 in terms of the coefficients of the polynomials and root extractions (square roots, cube roots, and fourth roots). The search for an analogue of the quadratic formula for the roots of all polynomials of degree 5 or higher was unsuccessful. In the 19th century, the reason for the failure to find such general formulas was explained by a subtle algebraic symmetry in the roots of a polynomial discovered by Evariste Galois. He found a way to attach a finite group to each polynomial $f(x)$, and there is an analogue of the quadratic formula for all the roots of $f(x)$ exactly when the group associated to $f(x)$ satisfies a certain technical condition that is too complicated to explain here. Not all groups satisfy the technical condition, and by this method Galois could give explicit examples of fifth degree polynomials, such as $x^5 - x - 1$, whose roots can't be described by anything like the quadratic formula. Learning about this application of group theory to formulas for roots of polynomials would be a suitable subject for a second course in abstract algebra.

## Structures in Modern Algebra

Fields, rings, and groups. We'll be looking at several kinds of algebraic structures this semester, the three major kinds being fields in chapter 2, rings in chapter 3, and groups in chapter 4, but also minor variants of these structures. We'll start by examining the definitions and looking at some examples. For the time being, we won't prove anything; that will come in later chapters when we look at those structures in depth. A note on notation. We'll use the standard notation for various kinds of numbers. The set of natural numbers, {0, 1, 2, . . .} is denoted N. The set of integers {. . . , −2, −1, 0, 1, 2, . . .} is denoted Z (for Zahlen, German for whole number). The set of rational numbers, that is, numbers of the form m n where m is an integer and n is a nonzero integer, is denoted Q (for "quotient"). The set of all real numbers, including all positive numbers, all negative numbers, and 0, is denoted R. And the set of complex numbers, that is, numbers of the form x + iy where x and y are real numbers and i 2 = −1, is denoted C.

## Fields

Informally, a field is a set equipped with four operations—addition, subtraction, multiplication, and division that have the usual properties. (They don't have to have the other operations that R has, like powers, roots, logs, and the myriad other functions like sin x.) Definition 1.1 (Field). A field is a set equipped with two binary operations, one called addition and the other called multiplication, denoted in the usual manner, which are both commutative and associative, both have identity elements (the additive identity denoted 0 and the multiplicative identity denoted 1), addition has inverse elements (the inverse of x denoted −x), multiplication has inverses of nonzero elements (the inverse of x denoted 1 x or x −1 ), multiplication distributes over addition, and 0 6= 1.

## Results and Findings

He concept of a group arose from the study of polynomial equations, starting with Évariste Galois in the 1830s, who introduced the term of *group* (*groupe*, in French) for the symmetry group of the roots of an equation, now called a Galois group. After contributions from other fields such as number theory and geometry, the group notion was generalized and firmly established around 1870. Modern group theory—an active mathematical discipline—studies groups in their own right.[a] To explore groups, mathematicians have devised various notions to break groups into smaller, better-understandable pieces, such as subgroups, quotient groups and simple groups. In addition to their abstract properties, group theorists also study the different ways in which a group can be expressed concretely, both from a point of view of representation theory (that is, through the representations of the group) and of computational group theory. A theory has been developed for finite groups, which culminated with the classification of finite simple groups, completed in 2004.[aa] Since the mid-1980s, geometric group theory, which studies finitely generated groups as geometric objects, has become an active area in group theory

One of the most familiar groups is the set of integers which consists of the numbers

..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...,[3] together with addition.

The following properties of integer addition serve as a model for the group axioms given in the definition below.

- For any two integers $a$ and $b$, the sum $a + b$ is also an integer. That is, addition of integers always yields an integer. This property is known as *closure* under addition.
- For all integers $a$, $b$ and $c$, $(a + b) + c = a + (b + c)$. Expressed in words, adding $a$ to $b$ first, and then adding the result to $c$ gives the same final result as adding $a$ to the sum of $b$ and $c$, a property known as *associativity*.
- If $a$ is any integer, then $0 + a = a + 0 = a$. Zero is called the *identity element* of addition because adding it to any integer returns the same integer.
- For every integer $a$, there is an integer $b$ such that $a + b = b + a = 0$. The integer $b$ is called the *inverse element* of the integer $a$ and is denoted $-a$.

A group is a set, $G$, together with an operation $\cdot$ (called the *group law* of $G$) that combines any two elements $a$ and $b$ to form another element, denoted $a \cdot b$ or $ab$. To qualify as a group, the set and operation, $(G, \cdot)$, must satisfy four requirements known as the *group axioms*

## Conclusion

The rough set theory has attracted attention of many researchers all over the world who contributed essentially to its development and application. In recent years we witnessed a rapid growth of rough set theory and its application world wide and extensive research has been carried out to compare the theory of rough sets with other theories of uncertainties. Among various branches of pure and applied mathematics, algebra was one of the first few subjects where the notion of rough set was applied. Some authors substituted an algebraic structure for the universal set and studied the roughness in algebraic structure. On the other hand, some authors studied the concept of rough algebraic structures. In 1994, Biswas and Nanda [10] introduced rough sets in the realm of group theory. From this publication onwards, a number of rough concepts related to algebraic structure have been introduced.

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