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APPLICATION OF TAGE ITERATIVE METHOD FOR THE NONLINEAR TWO-POINT BOUNDARY VALUE PROBLEMS

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Abstract

In this paper, utilizing three variable work focuses we have talked about a productive third arrange method for the arrangement of nonlinear integro-differential condition and the use of TAGE and Newton-TAGE iterative methods proposed by Evans. Since these TAGE methods are express in nature and coupled minimally, they are reasonable for use on parallel PCs. In this paper, we give deduction of scientific method in subtle elements. We examine the utilization of TAGE and Newton-TAGE iterative methods for the arrangement of linear and nonlinear integro-differential condition. Blunder investigation of the method is additionally talked about in subtle elements. We analyze the numerical outcomes acquired by the proposed iterative methods with the comparing progressive over unwinding (SOR) and Newton-SOR iterative methods.

1. INTRODUCTION

A few physical issues are portrayed numerically by linear and nonlinear integro differential equations. These equations emerge normally in various fields of material science, liquid elements, natural models, synthetic energy, for example, electric circuit investigation, disseminating hypothesis, colloidal scattering and many body issues. Most of the physically critical integro-differential equations can't be fathomed logically. So it is required to get productive numerical methods whose arrangements are of incredible significance to specialists and researchers. There has as of late been much consideration gave to the look for better numerical method for deciding an answer for both linear and nonlinear models. In 1984, Jain et al. have determined variable work methods for the arrangement of two-point nonlinear boundary value issues, in any case, their methods are not relevant to differential equations with singular coefficients. Of late, Grossmann et al have given different methods for the numerical arrangement of two-point boundary value issues.

2. DEVELOPMENT OF THE METHOD

We discretize the solution region $[0, 1]$ with the non-uniform mesh to such an extent that $0 < x_0 < x_1 \ldots \ldots \ldots < x_{N+1} = 1$ Our method comprises of three lattice focuses, x_1, x_{l+1} and x_{i-1} where $x_l - x_{l-1} = h$ and$x_{l+1} - x_1 = h_{l+1}$. Grid points focuses are given by $x_1 - x_0 + \sum_{l=1}^{i} h_1, i = 1(1)N + 1$. The mesh proportion $\sigma_1 = {}^{h_{l+1}}/_{h_1}$.At the point when $\sigma_1 = 1$, our method diminishes to the steady mesh case. The standard 5-point discretization of integro-differential condition is acquired utilizing third arrange variable mesh approximations for y' what's more,y'' . This requires the utilization of invented focuses outside the solution. This requires the utilization of invented focuses outside the solution region. The third request variable mesh method, which we present here, is in light of just three grid focuses$x_l, x_{l\pm1}$. This implies no invented focuses for fusing the boundary conditions are required.

Give the correct solution of y at the matrix a chance to point x_l be signified by $Y_1 = y(x_1)$what's more, y_1 be the inexact value of Y_l.

All through our dialog, we think about N as odd, i.e. our solution region contains odd number of interior lattice focuses.

APPLICATION OF TAGE ITERATIVE ALGORITHM

Incorporating the boundary conditions $y_0 = A$ and $y_{N+1} = B,$ we can re-write the linear difference equation in matrix form:

$$(A_1 + A_2)y = RH \qquad (1)$$

$$A_1 = \begin{bmatrix} b_1 & c_1 & & & & \boldsymbol{0} \\ a_2 & b_2 & & & & \\ & \ddots & \ddots & & & \\ & & & b_{N-2} & c_{N-2} & \\ & & & a_{N-1} & b_{N-1} & \\ \boldsymbol{0} & & & & & b_N \end{bmatrix}_{N\times N} \quad A_2 = \begin{bmatrix} b_1 & & & & & \boldsymbol{0} \\ & b_2 & c_2 & & & \\ & a_3 & b_3 & & & \\ & & \ddots & \ddots & & \\ & & & & b_{N-1} & c_{N-1} \\ \boldsymbol{0} & & & & a_N & b_N \end{bmatrix}_{N\times N}$$

$$\boldsymbol{y} = [y_1, y_2, \dots, y_N]' \text{ and } \boldsymbol{RH} = [J_1 - a_1 A, J_2, \dots, J_N - c_N B]' \equiv [RH_1, RH_2, \dots, RH_N]'$$

Then the TAGE iterative method (see Evans) may be written as:

$$(\boldsymbol{A}_1 + \omega_1 \boldsymbol{I})\boldsymbol{y}^* = \boldsymbol{RH} - (\boldsymbol{A}_2 - \omega_1 \boldsymbol{I})\boldsymbol{y}^{(s)}, \qquad s \geq 0 \tag{2a}$$

$$(\boldsymbol{A}_2 + \omega_2 \boldsymbol{I})\boldsymbol{y}^{(s+1)} = \boldsymbol{RH} - (\boldsymbol{A}_1 - \omega_2 \boldsymbol{I})\boldsymbol{y}^*, \qquad s \geq 0 \tag{2b}$$

Where y^* is an intermediate vector.

Let us denote $d_1 = b_1 + \omega_1$, $e_1 = b_1 - \omega_1, p_1 = b_1 + \omega_2$ and $q_1 = b_1 - \omega_2$, then the TAGE method (2a) and (2b) in matrix form may be written as

Since $(A_1 + \omega_1 I)$ and $(A_2 + \omega_2 I)$ are non-singular, hence we can write the TAGE method in the explicit form

$$\boldsymbol{y}^* = (\boldsymbol{A}_1 + \omega_1 \boldsymbol{I})^{-1}[\boldsymbol{RH} - (\boldsymbol{A}_2 - \omega_1 \boldsymbol{I})\boldsymbol{y}^{(s)}], \qquad s \geq 0 \tag{3a}$$

$$\boldsymbol{y}^{(s+1)} = (\boldsymbol{A}_2 + \omega_2 \boldsymbol{I})^{-1}[\boldsymbol{RH} - (\boldsymbol{A}_1 - \omega_2 \boldsymbol{I})\boldsymbol{y}^*], \qquad s \geq 0 \tag{3b}$$

Simplifying (3a) and (3b), we obtain the following algorithms:

$Sweep$-I: For $l = 1(2)N-2$, let $\Delta_1 = d_l d_{l+1} - c_l a_{l+1} \neq 0$

$$R_1 = RH_l - a^* a_l y_{l-1}{}^{(s)} - e_l y_l{}^{(s)}$$
$$R_2 = RH_{l+1} - e_{l+1} y_{l+1}{}^{(s)} - c_{l+1} y_{l+2}{}^{(s)}$$

Where $a^* = \begin{vmatrix} 0, & \text{if} \quad l = 1 \\ 1, & \text{otherwise} \end{vmatrix}$

Then $y_1^* = {(R_1 d_{l+1} - R_2 c_1)}/{\Delta_1}$ (3c)

$y_{l+1}^* = {(R_2 d_l - R_1 a_{l+1})}/{\Delta_1}$ (4a)

Finally for $l = N$, we have

$$y_N^* = (RH_N - a_N y_{N-1}{}^{(s)} - e_N y_N{}^{(s)}) / d_N$$

Sweep –II: For $l = 1$, we have $y_1{}^{(s+1)} = (RH_1 - q_1 y_1{}^* - c_1 y_2{}^*) / p_1$

For $l = 2(2)N - 1$, let $\Delta_2 = p_1 p_{l+1} - c_1 a_{l+1} \neq 0$

$$R_3 = RH_l - a_l y_{l-1}{}^* - q_l y_l{}^*$$

$$R_4 = RH_{l+1} - q_{l+1} y_{l+1}{}^* - b^* c_{l+1} y_{l+2}{}^*$$

Where $b^* = \begin{vmatrix} 0, & \text{if} \quad l = N-1 \\ 1, & \text{otherwise} \end{vmatrix}$

Then,

$$y_l{}^{(s+1)} = (R_3 p_{l+1} - R_4 c_l) / \Delta_2, \quad s \geq 0 \qquad \text{(4b)}$$

$$y_{l+1}{}^{(s+1)} = (R_4 p_l - R_3 a_{l+1}) / \Delta_2, \quad s \geq 0 \qquad \text{(4c)}$$

4. CONVERGENCE ANALYSIS

In this area we give the convergence analysis of the TAGE iterative method (3a) and (3b).

Since A_1 and A_2 are just made out of sub-matrices and single diagonal sections, we can undoubtedly assess their eigenvalues.

The eigenvalues λ of A_1 are given by

$\lambda = b_n$

And $(\lambda - b_k)(\lambda - b_{k+1}) - y_k^2 = 0, k = 1(2)N - 2$ (5)

Where $Y_k = \sqrt{a_{k+1} + c_k} > 0.$

Let the range of λ be given by $\bar{a} \leq \lambda \leq \bar{b}$.

The eigen values η of A_2 are given by

$\eta = b_1$

And $(\eta - b_k)(\eta - b_{k+1}) - y_k^2 = 0, k = 2(2)N - 1$ (6)

Let the range of η be given by $\bar{c} \leq \eta \leq \bar{d}$.

Combining (3.4.2a) and (3.4.2b), we have the TAGE iterative method

$y^{(s+1)} = Gy^s + g$ (7)

Where

$$\boldsymbol{G} = (\boldsymbol{A}_2 + \omega_2 \boldsymbol{I})^{-1}(\boldsymbol{A}_1 - \omega_2 \boldsymbol{I})(\boldsymbol{A}_1 + \omega_1 \boldsymbol{I})^{-1}(\boldsymbol{A}_2 - \omega_1 \boldsymbol{I})$$

$$\boldsymbol{g} = (\boldsymbol{A}_2 + \omega_2 \boldsymbol{I})^{-1}[\boldsymbol{I} - (\boldsymbol{A}_1 - \omega_2 \boldsymbol{I})(\boldsymbol{A}_1 + \omega_1 \boldsymbol{I})^{-1}]\boldsymbol{RH}$$

It is clear that the TAGE iterative method (4a) and (4b) or (5) converges to the exact solution $y = (A_1 + A_2)^{-1}RH$ if and only if the spectral radius $S(G)$of the iteration matrix G is less than unity.

Now, we study the convergence of the TAGE method which is governed by the norms of the matrix G.

We show that$S(G) < 1$ for any $\omega_1 > 0$ and $\omega_2 > 0$

Let D be a diagonal matrix given by

$D = diag(1, d_2, d_3, \ldots \ldots, d_N)$ (8)

Where $d_M = \sqrt{\frac{c_1 c_2 \ldots\ldots c_{M-1}}{a_2 a_3 \ldots\ldots a_M}}$ $M = 2(1)N,$ $c_N a_{N+1} > 0$

This implies $D^{-1} = \text{diag}(1, \frac{1}{d_2}, \frac{1}{d_3}, \ldots \ldots, \frac{1}{d_N})$ (9)

Where $\frac{1}{d_M} = \sqrt{\frac{a_2 a_3 \ldots\ldots a_M}{c_1 c_2 \ldots\ldots c_{M-1}}}$: $M = 2(1)N.$

Consider the matrices A_1^* and A_1^* which are similar to A_1 and A_2 respectively and are given by

By the help of (2) and (3) from (4a) and (4b), we obtain

$A_1^* = DA_1D^{-1}$ (10a)

$A_2^* = DA_2D^{-1}$ (10b)

5. COMPUTATIONAL RESULTS

We have tackled the accompanying benchmark issues on both uniform and non-uniform work whose correct solutions are known to us and contrasted the outcomes and the comparing successive over relaxation (SOR) and Newton-SOR method (see Hageman and Young). For uniform work we take $\sigma_1 = \sigma = 1$ furthermore, for non-uniform work, we take $\sigma_1 = \sigma = a\ constant\ (\neq 1), l = 1(1)N + 1$. At that point the estimation of first work dividing on the left is

$$h_1 = \frac{(1-\sigma)}{(1-\sigma^{N+1})}, \quad \sigma \neq 1 \qquad (11)$$

Hence, for non-uniform work, given the value of N and σ, we can figure h_1 from the above relation and the rest of the work is resolved from $h_{l+1} = \sigma h_1, l = 1(1)N.$

For example: Let N = 11 (odd number of internal mesh points)

Let $\sigma = 0.8$ then $h_1 = \frac{(1-0.8)}{(1-0.8^{12})}$

and consequently $h_2 = \sigma h_1, h_3 = \sigma h_2, h_4 = \sigma h_3, \ldots, h_{12} = \sigma h_{11}$.

While this sort of restriction isn't material to uniform work case. By taking$\sigma_1 = \sigma = 1$, we can apply techniques (10), (2) and (11) specifically to uniform work. All calculations were completed in MATLAB figuring dialect. In all cases, we have thought about $y^{(0)} = 0$ and the iterations were ceased when the supreme blunder resistance $\left|y^{(s+1)} - y^{(s)}\right| \leq 10^{-10}$ was accomplished.

Example 3.1: (Linear Singular Problem)

$$y'' + \frac{\alpha}{x} y' - \frac{\alpha}{x^2} y = \alpha \sinh x + (4+\alpha) x \cosh x + \int_0^1 \left[6\left(\frac{xs}{1+s^3}\right)^2 \sinh x + 2x\cosh(xs) \right] ds,$$

$$0 < x < 1, 0 < s < 1$$

The boundary values are given by$y(0) = 0, y(1) = \frac{(e-e^{-1})}{2}$

The correct solution of the issue is given by is given by$y(x) = x^2 sinhx$.The root mean square (RMS) mistakes, optimal values $\omega_{opt}, \omega_{1opt}, \omega_{2opt}$ of relaxation parameters and number of iterations both for TAGE and SOR methods are organized in Table 1(A) for $(\propto, \sigma) = (1, 1.1)$ what's more $(\propto, \sigma) = (2,0.9)$, and in Table 1(B) for and , separately.

Table 1(A)

Example-1: The RMS Errors (non uniform mesh case)

$\propto = 1, \sigma = 1.1$

N	SOR			TAGE				RMS errors
	ω_{opt}	Iter	cputime	ω_{1opt}	ω_{2opt}	Iter	cputime	
11	1.497	45	0.003402 s	0.590	0.598	35	0.003024 s	0.6230(-04)
21	1.667	73	0.006138 s	0.340	0.346	60	0.003830 s	0.3030(-05)
31	1.733	103	0.006928 s	0.234	0.240	81	0.005024 s	0.6527(-06)
41	1.764	120	0.011499 s	0.187	0.193	94	0.005821 s	0.3494(-06)
51	1.786	124	0.016106 s	0.175	0.181	101	0.006264 s	0.2740(-06)
61	1.797	133	0.017726 s	0.138	0.139	108	0.008906 s	0.2411(-06)
71	1.803	151	0.021307 s	0.144	0.146	109	0.009274 s	0.2207(-06)
81	1.811	168	0.024428 s	0.144	0.150	116	0.011311 s	0.2057(-06)

$$\propto = 2, \sigma = 0.9$$

N	SOR			TAGE				RMS errors
	ω_{opt}	Iter	cputime	ω_{1opt}	ω_{2opt}	Iter	cputime	
11	1.503	48	0.003626 s	0.530	0.538	39	0.002772 s	0.4146(-03)
21	1.693	92	0.006926 s	0.296	0.304	73	0.003748 s	0.1527(-03)
31	1.775	136	0.010519 s	0.206	0.212	108	0.005672 s	0.1025(-03)
41	1.817	176	0.013442 s	0.165	0.161	135	0.006666 s	0.8329(-04)
51	1.844	196	0.019789 s	0.134	0.138	163	0.008140 s	0.7295(-04)
61	1.859	214	0.037378 s	0.117	0.123	180	0.009037 s	0.6617(-04)
71	1.873	236	0.035354 s	0.102	0.106	198	0.010003 s	0.6116(-04)
81	1.881	264	0.040698 s	0.097	0.101	214	0.012218 s	0.5720(-04)

Non-uniform mesh case

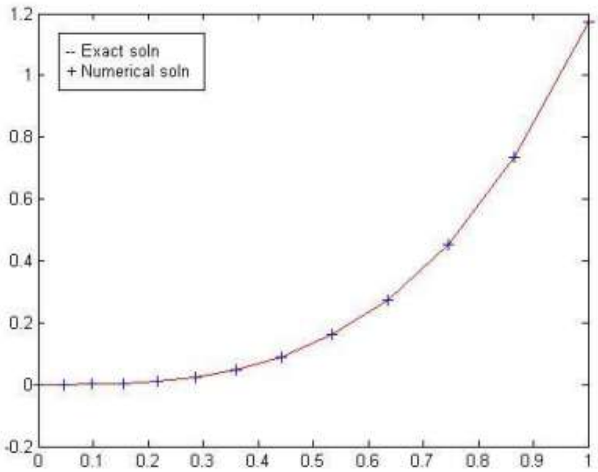

Fig 1(a) Comparison of plots of solution

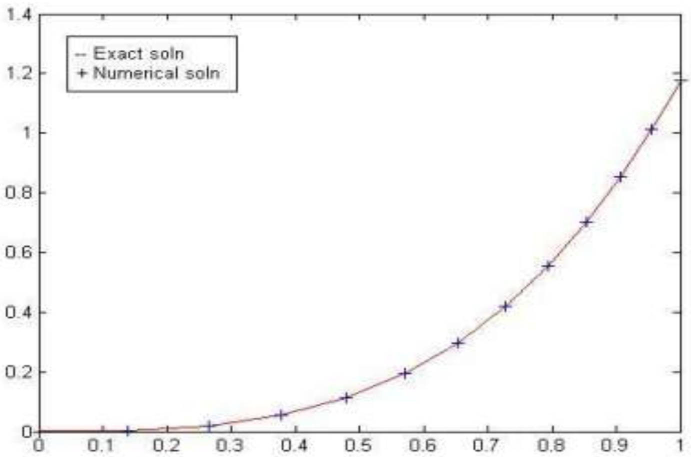

Fig 1(b) Comparison of plots of solution

CONCLUSION

We have introduced another three point variable work technique for exactness for the solution of second request nonlinear two point limit value issue with driving capacities in integral frame. Be that as it may, for the proposed technique lessens to a constant work strategy. The proposed strategy is pertinent when the quantity of inward network points of the solution space is odd. This technique is effectively connected to linear and nonlinear issues with particular coefficients.

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