WAVE PROPAGATION IN POROUS LAYERED MEDIUM

Rajkala*

Abstract

In this paper, the various characteristics of the propagation of surface waves, in fluid saturated incompressible porous layered media, are discussed. Model of a fluid saturated porous half-space lying under two layers of different liquids. The first layer is taken to be non-homogeneous with non-homogeneity varies with depth and the second layer to be homogeneous.

1.1 INTRODUCTION

The study of surface wave propagation in saturated porous media is of practical significance in the field of seismic engineering, because most of the geological materials can be grouped into a certain family of porous media. In particular, the portion of these media as structure foundation is mostly composed of saturated porous media as water saturated soil deposit. The Biot(1941) model of porous media has been extensively used by many researchers to study the surface wave propagation in fluid saturated porous media. Rao and Sharma (1978) studied the Love wave propagation in poroelasticity by considering the porous medium with a layer of different porous medium over it. Tajuddin and Lingam (1990) discussed the propagation of surface wave in a poroelastic solid layer lying over an elastic solid. Sharma, Kumar and

* Assist. Prof. Dr. BRA. Govt. College, Kaithal-136027(Haryana), India

Gogna (1991) discussed the surface wave propagation in a liquid saturated porous layer lying over a homogeneous transversely isotropic elastic half-space and under a uniform layer of liquid.

Based on the theory of porous media, which is based on the Fillunger model, de Boer and Liu (1994) investigated the problem of plane waves in a semi-infinite fluid saturated incompressible porous medium and discussed the dispersion relationship and the attenuation. They observed that the propagation of transverse waves in the fluid phase is completely due to the interaction between the two phases and pore pressure is produced in the process of reflection, even in the case of the incidence of transverse wave.Liu and de Boer (1997) discussed the wave propagation characteristics including dispersion and attenuation of Rayligh and Love type waves in a fluid saturated porous medium where by taking the porous medium to be consist of a microscopically incompressible porous solid skeleton saturated by an microscopically incompressible liquid.

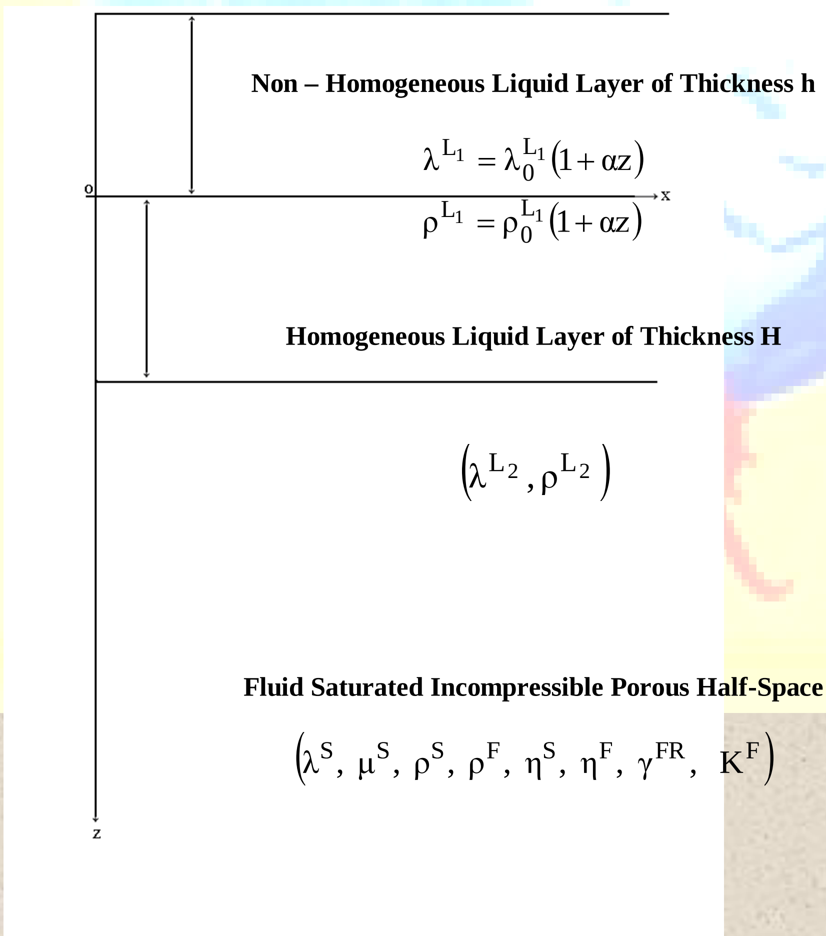
In this paper, the various characteristics of the propagation of surface waves, in fluid saturated incompressible porous layered media, are discussed. Model considered in problem-I consist of a fluid saturated incompressible porous half-space lying under two layers of different liquids. The first layer is taken to be non-homogeneous with non-homogeneity varies with depth and the second layer to be homogeneous. The Problem -II is devoted to the dispersion and attenuation of Rayleigh-type surface waves, in a model consisting of a poroelastic plate of a fluid saturated incompressible porous material lying between an empty porous elastic layer and an empty porous elastic half-space.

Problem-I

Oceanic models involving double liquid layer of inhomogeneous and homogeneous liquids lying over the fluid saturated soil and other materials, which are porous but incompressible in nature, are often present in the earth model. So in the present part of this chapter, we have investigated the surface wave propagation in one such model by taking first layer to be non – homogeneous, with non–homogeneity varying with depth, and the second layer homogeneous. The half – space is taken as a two-phase system with incompressible solid phase and an incompressible fluid phase. Frequency equation relating the phase velocity with the wave number and other material parameters is derived and the variations of phase velocity with wave number for different values of the non-homogeneity parameter and for different values of the ratio of the thicknesses of layers are presented graphically and are discussed. Depending upon the thicknesses of the layers, some particular cases have also been included.

FORMULATION OF THE PROBLEM AND ITS SOLUTION

We consider a model consisting of a double layer of two different liquids, resting on a half – space of a fluid saturated incompressible porous medium of infinite extent. The upper layer L_1 is non-homogeneous and is of thickness h, whereas the lower layer L_2 is homogeneous and its thickness is taken to be H. The coordinate system is selected with xy-plane coinciding with the interface between two layers, x-axis along the length and z-axis perpendicular to the interface along the direction of increasing depth. So the layers L_1, L_2 and the half-space occupy the regions $-h < z \leq 0$, $0 \leq z < H$ and $z \geq H$ respectively as shown in Fig.1



Geometry of the Investigated Problem

Fig. 1

We also consider the waves of plane strain with wave front parallel to the y-axis so that the field components in the y-direction vanish and are independent of y coordinate. So we have

$$\left.\begin{aligned}\mathbf{u_S} &= \left(u^S, 0, w^S\right),\\ \mathbf{u_F} &= \left(u^F, 0, w^F\right).\end{aligned}\right\} \tag{1.1}$$

Using for the half-space in the absence of body forces take the form

$$(\lambda^S+\mu^S)\frac{\partial\theta^S}{\partial x}+\mu^S\nabla^2u^S-\eta^S\frac{\partial p}{\partial x}-\rho^S\frac{\partial^2u^S}{\partial t^2}+S_V\left(\frac{\partial u^F}{\partial t}-\frac{\partial u^S}{\partial t}\right)=0, \tag{1.2}$$

$$(\lambda^S+\mu^S)\frac{\partial\theta^S}{\partial z}+\mu^S\nabla^2w^S-\eta^S\frac{\partial p}{\partial z}-\rho^S\frac{\partial^2w^S}{\partial t^2}+S_V\left(\frac{\partial w^F}{\partial t}-\frac{\partial w^S}{\partial t}\right)=0, \tag{1.3}$$

$$\eta^F\frac{\partial p}{\partial x}+\rho^F\frac{\partial^2u^F}{\partial t^2}+S_V\left(\frac{\partial u^F}{\partial t}-\frac{\partial u^S}{\partial t}\right)=0, \tag{1.4}$$

$$\eta^F\frac{\partial p}{\partial z}+\rho^F\frac{\partial^2w^F}{\partial t^2}+S_V\left(\frac{\partial w^F}{\partial t}-\frac{\partial w^S}{\partial t}\right)=0, \tag{1.5}$$

$$\eta^S\left(\frac{\partial^2u^S}{\partial x\partial t}+\frac{\partial^2w^S}{\partial z\partial t}\right)+\eta^F\left(\frac{\partial^2u^F}{\partial x\partial t}+\frac{\partial^2w^F}{\partial z\partial t}\right)=0, \tag{1.6}$$

$$\tau_{zz} = \lambda^S \theta^S + 2\mu^S \frac{\partial w^S}{\partial z}, \quad (1.7)$$

$$\tau_{xz} = \mu^S \left(\frac{\partial u^S}{\partial z} + \frac{\partial w^S}{\partial x} \right). \quad (1.8)$$

where

$$\theta^S = \frac{\partial u^S}{\partial x} + \frac{\partial w^S}{\partial z}. \quad (1.9)$$

For the layers L_1 and L_2, equations governing the motions of liquids are given by Ewing, Jardetzky and Press (1957) as

$$\lambda^{L_i} \nabla(\nabla \mathbf{u}_{L_i}) = \rho^{L_i} \frac{\partial^2 \mathbf{u}_{L_i}}{\partial t^2}, \quad (1.10)$$

$$\tau_{m_0 n_0}^{L_i} = \lambda^{L_i} \nabla \mathbf{u}_{L_i} \delta_{m_0 n_0}, \quad (1.11)$$

where m_0, n_0 =1, 2, 3 and i = 1, 2 refer to layers L_1 and L_2 respectively. In these two equations $\mathbf{u}_{L_i}$ are the displacement vectors, λ^{L_i} are the bulk modulii of the liquids, ρ^{L_i} are their densities and $\tau_{m_0 n_0}^{L_i}$ are the components of the stresses in the liquids. For the present problem, the displacement vectors $\mathbf{u}_{L_i} = \left(u^{L_i}, 0, w^{L_i}\right)$. So the equations (4.10) and (4.11) for the layers L_1 and L_2 are simplified as:

$$\frac{\partial}{\partial x}\left(\lambda^{L_1} \theta^{L_1}\right) = \rho^{L_1} \frac{\partial^2 u^{L_1}}{\partial t^2}, \quad (1.12)$$

$$\frac{\partial}{\partial z}\left(\lambda^{L_1}\theta^{L_1}\right)=\rho^{L_1}\frac{\partial^2 w^{L_1}}{\partial t^2}, \quad (1.13)$$

$$\lambda^{L_2}\frac{\partial\theta^{L_2}}{\partial x}=\rho^{L_2}\frac{\partial^2 u^{L_2}}{\partial t^2}, \quad (1.14)$$

$$\lambda^{L_2}\frac{\partial\theta^{L_2}}{\partial z}=\rho^{L_2}\frac{\partial^2 w^{L_2}}{\partial t^2}, \quad (1.15)$$

$$\tau_{zz}^{L_i}=\tau_{xx}^{L_i}=\lambda^{L_i}\theta^{L_i}, \quad (1.16)$$

where

$$\theta^{L_i}=\frac{\partial u^{L_i}}{\partial x}+\frac{\partial w^{L_i}}{\partial z}. \quad (1.17)$$

Let the non-homogeneity of the layer L_1 be taken as

$$\lambda^{L_1}=\lambda_0^{L_1}(1+\alpha z), \qquad \rho^{L_1}=\rho_0^{L_1}(1+\alpha z), \quad (1.18)$$

where $\lambda_0^{L_1}$ and $\rho_0^{L_1}$ are the bulk modulus and density at the surface z = 0 and α is the non-homogeneity parameter of the non-homogeneous layer. For further considerations, it is convenient to introduce in equations (4.2) – (4.17), the dimensionless quantities defined as

$$x'=\frac{\omega^*}{c_1}x, \quad z'=\frac{\omega^*}{c_1}z, \quad t'=\omega^* t, \quad u'^S=\left(\frac{\lambda^S+2\mu^S}{E}\right)\frac{\omega^*}{c_1}u^S, \quad w'^S=\left(\frac{\lambda^S+2\mu^S}{E}\right)\frac{\omega^*}{c_1}w^S,$$

$$u'^F=\left(\frac{\lambda^S+2\mu^S}{E}\right)\frac{\omega^*}{c_1}u^F, \quad w'^F=\left(\frac{\lambda^S+2\mu^S}{E}\right)\frac{\omega^*}{c_1}u^F, \quad p'=\frac{p}{E}, \quad \tau'_{xz}=\frac{\tau_{xz}}{E}, \quad \tau'_{zz}=\frac{\tau_{zz}}{E}$$

$$u'^{L_1} = \left(\frac{\lambda^S + 2\mu^S}{E}\right)\frac{\omega^*}{c_1}u^{L_1} \quad , \qquad w'^{L_1} = \left(\frac{\lambda^S + 2\mu^S}{E}\right)\frac{\omega^*}{c_1}w^{L_1} \quad , \qquad \tau'^{L_1}_{zz} = \frac{\tau^{L_1}_{zz}}{E} \quad ,$$

$$u'^{L_2} = \left(\frac{\lambda^S + 2\mu^S}{E}\right)\frac{\omega^*}{c_1}u^{L_2}, \quad w'^{L_2} = \left(\frac{\lambda^S + 2\mu^S}{E}\right)\frac{\omega^*}{c_1}w^{L_2}, \quad \tau'^{L_2}_{zz} = \frac{\tau^{L_2}_{zz}}{E} \qquad (1.19)$$

where ω^* is a constant having the dimensions of frequency, E is the Young's modulus of elasticity and c_1 is given by (2.17). Using (4.19) in equations (4.2) – (4.9), (4.12) – (4.17), with the aid of (4.18) and after suppressing the primes, we get the following dimensionless form of the governing equations

$$(1\text{-}\delta^2)\frac{\partial\theta^S}{\partial x} + \delta^2\nabla^2 u^S - \eta^S\frac{\partial p}{\partial x} - \delta_1^2\frac{\partial^2 u^S}{\partial t^2} + \delta_2\left(\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right) = 0, \qquad (1.20)$$

$$(1\text{-}\delta^2)\frac{\partial\Theta^S}{\partial z} + \delta^2\nabla^2 w^S - \eta^S\frac{\partial p}{\partial z} - \delta_1^2\frac{\partial^2 w^S}{\partial t^2} + \delta_2\left(\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right) = 0, \qquad (1.21)$$

$$\eta^F\frac{\partial p}{\partial x} + \frac{\rho^F}{\rho^S}\delta_1^2\frac{\partial^2 u^F}{\partial t^2} + \delta_2\left(\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right) = 0, \qquad (1.22)$$

$$\eta^F\frac{\partial p}{\partial z} + \frac{\rho^F}{\rho^S}\delta_1^2\frac{\partial^2 w^F}{\partial t^2} + \delta_2\left(\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right) = 0, \qquad (1.23)$$

$$\eta^S\left(\frac{\partial^2 u^S}{\partial x\partial t} + \frac{\partial^2 w^S}{\partial z\partial t}\right) + \eta^F\left(\frac{\partial^2 u^F}{\partial x\partial t} + \frac{\partial^2 w^F}{\partial z\partial t}\right) = 0, \qquad (1.24)$$

$$\tau_{zz} = \left(1\text{-}2\delta^2\right)\frac{\partial u^S}{\partial x} + \frac{\partial w^S}{\partial z}, \qquad (1.25)$$

$$\tau_{xz} = \delta^2\left(\frac{\partial u^S}{\partial z} + \frac{\partial w^S}{\partial x}\right), \qquad (1.26)$$

$$\frac{\partial \theta^{L_1}}{\partial x} = \delta_{L_1}^2 \frac{\partial^2 u^{L_1}}{\partial t^2}, \qquad (1.27)$$

$$\frac{\partial \theta^{L_1}}{\partial z} + \frac{\alpha}{(1+\alpha z)}\theta^{L_1} = \delta_{L_1}^2 \frac{\partial^2 w^{L_1}}{\partial t^2}, \qquad (1.28)$$

$$\frac{\partial \theta^{L_2}}{\partial x} = \delta_{L_2}^2 \frac{\partial^2 u^{L_2}}{\partial t^2}, \qquad (1.29)$$

$$\frac{\partial \theta^{L_2}}{\partial z} = \delta_{L_2}^2 \frac{\partial^2 u^{L_2}}{\partial t^2}, \qquad (1.30)$$

where

$$\delta = \frac{\beta_0}{c_0}, \delta_1 = \frac{c_1}{c_0}, \ \beta_0 = \sqrt{\frac{\mu^S}{\rho^S}}, \ c_0 = \sqrt{\frac{\lambda^S + \mu^S}{\rho^S}}, \ \delta_{L_1} = \frac{c_1}{\alpha_{L_1}}, \ \delta_{L_2} = \frac{c_1}{\alpha_{L_2}},$$

$$\alpha_{L_1} = \sqrt{\frac{\lambda_0^{L_1}}{\rho_0^{L_1}}}, \ \alpha_{L_2} = \sqrt{\frac{\lambda^{L_2}}{\rho^{L_2}}} \text{ and } \delta_2 = \frac{S_V \ c_1^2}{\omega' \rho^S c_0^2}. \qquad (1.31)$$

BOUNDARY CONDITIONS

The boundary conditions, in dimensionless form, for the present problem are as follows:

(a) The free surface of the liquid layer L_1, which is the vanishing of the normal stress component at z = - h, i.e.

$$\tau_{zz}^{L_1} = 0. \qquad (1.32)$$

(b) Conditions at the interface between two liquid layers are the continuity of normal stress and displacement components. So at the interface z = 0, we have

$$\left.\begin{array}{l}\text{(i) } \tau_{zz}^{L_1} = \tau_{zz}^{L_2}, \\ \text{(ii) } w^{L_1} = w^{L_2}.\end{array}\right\} \qquad (1.33)$$

(c) At the interface z = H, continuity of normal stress components, vanishing of shear component of stress and the continuity of normal displacement components, i.e.

$$\left.\begin{array}{l}\text{(i) } \tau_{zz}^{L_2} = \tau_{zz} - p, \\ \text{(ii) } \tau_{xz} = 0, \\ \text{(iii) } w^{L_2} = w^{S}.\end{array}\right\} \qquad (1.34)$$

4.6 NUMERICAL RESULTS AND DISCUSSIONS

With a view toward illustrating the analytical procedure presented in the preceding sections, we consider an example for numerical discussion and consider a model for which the physical constants for half pace are taken as defined in Chapter-2, where as for liquid layers following Ewing, Jardetzky and Press (1957), we have $\lambda_0^{L_1} = \lambda^{L_2} = 0.214 \times 10^{10}\,\text{N/m}^2$ and $\rho^{L_0^1} = \rho^{L_2} = 1.0 \times 10^3\,\text{kg/m}^3$.

The phase velocity c as a function of wave number k and various physical parameters in complex form, showing that the waves are attenuated in space. If we write

$$\frac{1}{c} = \frac{1}{v} + i\frac{q}{\omega} \qquad (1.35)$$

so that the wave-number k = R_1 + iq, where $R_1 = \frac{\omega}{v}$, v and q are real numbers. This shows that v is the propagation speed, R_1 is the real wave-number and q is the attenuation coefficient of the waves.

The results depict the variation of phase velocity with respect to wave number and their graphical representations . It is evident that for any value of αh, starting from a higher value, the phase velocity falls very quickly to some lower value then decreases gradually to the velocity of the waves of short wave length. The significant fall at the vanishing wave number is due to the damping effect of the overlying liquid layers and also the viscous damping caused by internal friction from the interaction mechanism between the skeleton and pore liquid present in the pores. So, in the beginning when wave number is small the waves are highly dispersive, their depressiveness decreases with increase of wave number and ultimately for all values of αh, the phase velocity is constant and hence the waves become non-dispersive. Curves are drawn for three different values of αh and increase in phase velocity with increase in αh indicates that the non-homogeneity of the liquid layer also affects the dispersive character of the waves. Variations are shown for two values of the ratio $\frac{H}{h}$ and we observe that this ratio affects the phase velocity only quantitatively, but not qualitatively. This is again justified when the curves are drawn for three different values of $\frac{H}{h}$ at some fixed values of αh .It is clear that the phase velocity increases with the decrease in the values of this ratio.