FLOW AND HEAT TRANSFER IN STRAIGHT DUCT

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ABSTRACT:

In the present work, a wall element technique based on finite element method (FEM) has been developed and adopted in the zone which is closed to the solid walls of a long straight channel which replaces the traditional use of the empirical laws for the determination of confined turbulent flow associated heat transfer. The validity of this technique was investigated and well compared with other standard techniques.

Keywords: - ***Turbulent flow, heat transfer, wall element technique, FEM.***

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I. Introduction:

Turbulent fluid flow and heat transfer phenomena has numerous important applications such as flow in the blade passages of turbo machinery, heat exchangers and cooling system. Due to the complexity of the equations which governing the fluid motion by the Navier-Stokes (N-S) equations and the heat transfer by the energy equations, an analytical solution is intractable and during the last three decades, attention has been focused on the numerical simulation of flow process, the so called computational fluid dynamics (CFD). Finite element methods [1-4] are one of the methods which established itself as a powerful tool, feasible, complementary and competitive alternative to other existing numerical methods. In dealing with confined turbulent, an effective technique is required to model the variation of the primitive variables in a zone close to a solid boundary, the near wall zone (N.W.Z.), incorrect modeling can affect the values of the primitive variables throughout the flow domain.

In order to accommodate the rapid transfer of shear and variations in velocities, turbulent kinetic energy and temperature, within this zone, attention has been paid to model this zone accurately to obtain correct an overall predictions particularly the transfer of shear from the solid wall, with associated large variations in velocity, turbulence kinetic energy, and temperature. If a conventional finite element is used to model the N.W.Z. then significant grid refinement would be required. Several solution procedures have been suggested in order to avoid such excessive spatial refinement [5-7]. A more common approach, widely adopted is to terminate the computational domain (main domain) at some small distance away from the wall, where the gradients of independent variables are relatively small, and an appropriate technique then used to model the flow behavior of the fluid and the heat in the N.W.Z. Traditionally, the concepts of universal laws [8, 9] are used, to depict the variables behavior in near wall zone. In the present work, these concepts have been replaced by adopting a finite elements technique based near wall zone. The validity of the adopted wall element technique was investigated and well compared with other standard technique.

II. Mathematical Model:

Equations which are commonly used to describe the momentum which implied by the Navier-Stokes equations and the mass conservation by the continuity equation govern steady - state two dimensional flow of an incompressible viscous Newtonian fluid with no body forces acting are, respectively;

$$\rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_e\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \quad \ldots\ldots\ldots\ldots (1)$$

and

$$\frac{\partial u_i}{\partial x_i} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where u_i, are the velocities with respect to an orthogonal Cartesian coordinate systems x_i (where i, j = 1, 2), p is the local pressure, ρ is the fluid density, μ_e is the effective viscosity which is given by $\mu_e = \mu + \mu_t$, μ_t are the turbulent viscosity and $\mu$is the dynamic viscosity of the fluid.

Equation (1) and (2) cannot be solved unless a turbulence closure model can be provided to evaluate the turbulent contribution to μ_e. The simplest model is via an algebraic formula [10] which has limited application and therefore this model is not adopted in the present work, but an alternative (Prandtl [11]-kolmogorov [12]) model is used. For the present work, a one-equation model has been adopted so that,

$$\mu_t = C_\mu \rho k^{1/2} 1_\mu \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where k is the turbulence kinetic energy, 1_μ is the length scale of turbulence which has been specified algebraically for the present purposes and C_μ is a constant, which is taken as 0.22. The distribution of k is depicted by the transport equation,

$$\rho u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + \mu_t \frac{\partial u_i}{\partial x_j}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] - B \quad ..(4)$$

Where B= $C_D \rho k^{3/2} / 1_\mu$, μ_t / σ_k is the turbulent diffusion coefficient, σ_k is the turbulent prandtl and C_D is a constant.

The temperature (T) distribution can be then obtained from an energy equation written in the form;

$$\rho u_j \frac{\partial T_i}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{\sigma}+\frac{\mu_t}{\sigma_t}\right)\frac{\partial T}{\partial xj}\right] \quad \text{……………..…….. (5)}$$

In which μ_t / σ_t, σ , σ_t are respectively, the thermal molecular diffusivity of the fluid, the laminar Prandtl number which is usually taken as 0.7 and the turbulent Prandtl number is given:

$\sigma_t = 0.7$ $\quad Y^+ \leq 5$

$\sigma_t = 1.4 – (0.7 (13-Y+))/8$ $\quad 5 < Y^+ \leq 13$

$\sigma_t = 1.4$ $\quad 13 < Y^+ \leq 17$

$\sigma_t = 0.95 + (0.45 (25-Y+))/8$ $\quad 17 < Y^+ \leq 25$

$\sigma_t = 0.95$ $\quad Y^+ > 25$

Where $Y^+ = (y \sqrt{(\tau_w \rho)}) / \mu$,τ_w is the wall shear stress, y is the normal distance measured away from the wall into the fluid. The above governing equations (1), (2), (4) and (5) are then solved within the computational domain (i.e. main domain) using a standard finite element method, in which the Galerkin weighted residual approach is adopted to solve the discretising equations. Green theorem is used to reduce the order of the equations to unify resulting in a 'weak formulation' which resulted in non-linear equation matrix which is solved then by using matching process. Pressure procedure developed by Schneider [13], which implements the conservation of mass through the use of the pressure. Within the near wall zone (N.W.Z.), either conventional finite element (i.e. 2-D elements up to the wall) can be used, however an excessive mesh refinement was needed which is expensive in computer time and memory, or universal laws to bridge from a solid boundary to the main domain (Figure 1). In the present work, a finite elements technique has been adopted, using one-dimensional normal to the wall (Figure 2). The validity of the wall element technique was investigated and well compared with other standard technique in along straight duct.

III. Boundary Conditions:

In the present work, pressure flow was considered. Fully developed Dirichlet conditions are assumed on all variables upstream. No slip condition were imposed on solid boundaries and tractions updated downstream. Tractions are given by,

$$\tau_{x_1} = -p + \frac{\mu_e}{\rho}\left(\frac{\partial u_1}{\partial x_1}\right) \qquad x_1\text{- parallel to walls}$$

$$\tau_{x_2} = \frac{\mu_e}{\rho}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) \qquad x_2\text{- normal to walls}$$

Zero gradients of turbulence kinetic energy as Neuman conditions are used at downstream. The updated traction technique, can be equally applied to the energy equation, and is given by;

$$\left(\frac{\mu}{\sigma} + \frac{\mu_t}{\sigma_t}\right)\frac{\partial T}{\partial x_1} \qquad x_1\text{- parallel to walls}$$

On solid boundary boundaries, no slip condition was imposed and a constant heat flux is specified by imposing the temperature gradient value with respect to normal direction on the wall.

IV. Results and Discussion:

The validity of the adopted wall element technique is tested by, analyzing fully developed turbulent flow and associated heat transfer in a long straight duct of width D, which is taken as 1.0 in the present work and length L. Different Reynold numbers of 12,000 and 50,000 based upon the width of D was investigated. Compatible fully developed velocity and kinetic energy profiles were imposed at the upstream section when fully developed turbulent flow was considered at the first stage and the tractions were updated at downstream. These profiles were obtained by using the outlet values form each iteration as new approximations to the values at the inlet until a convergent condition is satisfied.

Convergent velocity profiles are presented in Figures 3 and 4. Clearly, the results obtained from the adoption of the presently advocated technique 1-D element normal to the wall exhibits excellent agreement with the correct solution which resulted from the complete mapping (i.e. 2-D element up to the wall). These are, superior to those obtained using universal laws. Figure 5 shows an excellent agreement between the adopted technique and experimental results [14]. Once more, the 'correct' values are remarkably close to those obtained from the advocated technique, as shown in Figure 6 refers to the kinetic energy. It is clearly from the results obtained, that the validity of the wall element technique (1-D elements in one direction) which has most advantageous owing to the number of elements used in the near wall zone are similar to those obtained from the use of 2-D element up to the wall which is not economic technique and needs excessive refinement when fully-developed turbulent flow was considered. The validity of the wall element technique (one-dimensional normal) has been tested and approved [15]. In the present work, this validity of the developed technique has been proved again when heat transfer is considered.

Temperature and velocity distributions were imposed at the duct inlet. Temperature values were those obtained after several solutions were undertaken [16-17], with constant heat flux boundary conditions at the solid walls, and downstream values of temperature re-imposed at the upstream for the following iteration. This follows the technique when updated tractions were used. This gave a smooth distribution of temperature compatible with the upstream flow conditions. Figure 7 shows that the adopted technique and complete mapping are almost identical whilst approximately 5% discrepancy, maximum, exists when comparing these to the temperature profile obtained using universal laws and also shows a good agreement with the experimental results [18]. Figure 8 shows a large difference in values when considering temperature profiles in the longitudinal direction.

Again, the advocated wall element technique has been demonstrated to be superior to that usually used and compares very favorably with an accurate, alternative calculation. This is primarily due to the inaccuracies in gradients of temperature when universal laws are used which reflects the same trend as when velocities are evaluated.

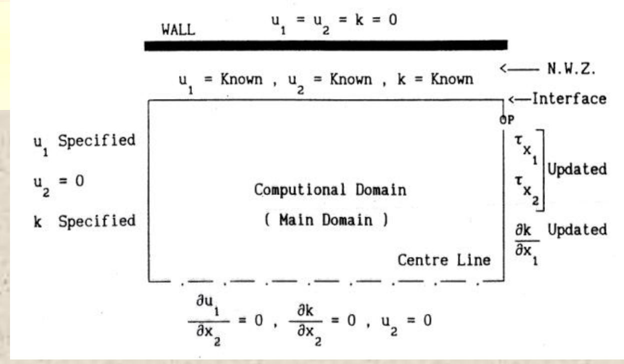

Figure 1: Boundary conditions when the mesh is terminated at small distance away from the wall.

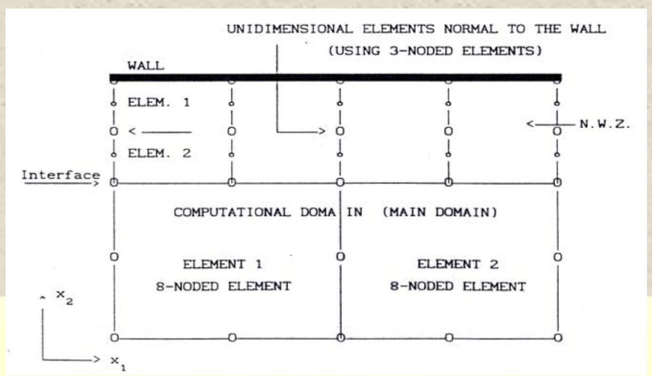

Figure 2: One-dimensional elements in one-direction normal to the wall used in the N.W.Z.

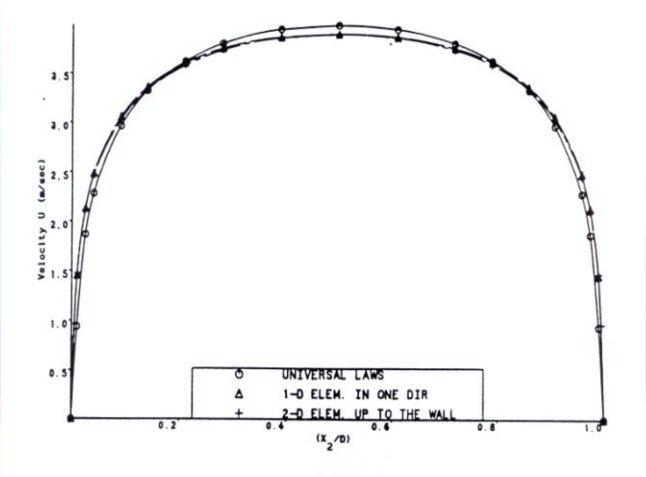

Figure 3: Turbulent velocity profiles for fully-developed flow, at 8D downstream, L=8D, Re=12.000

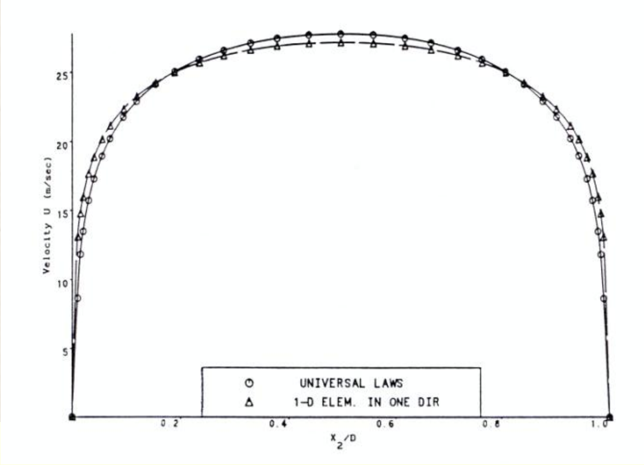

Figure 4: Turbulent velocity profiles for fully-developed flow, at 8D downstream, L=8D, Re=50.000

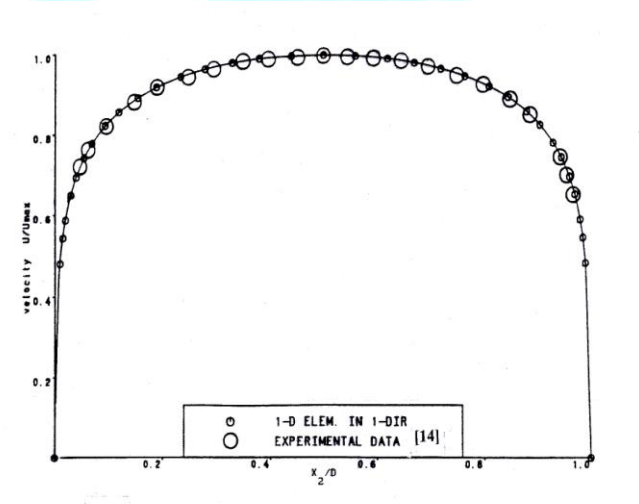

Figure 5: Turbulent velocity profiles for fully-developed flow, at 8D downstream, L=8D, Re=50.000

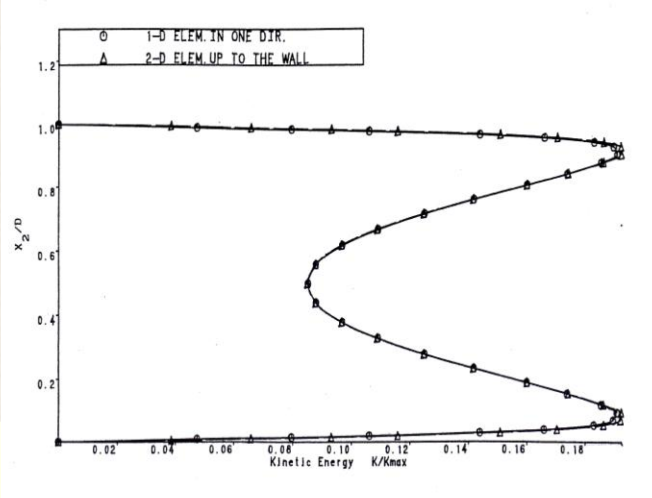

Figure 6: Fully-developed kinetic energy profiles for turbulent flow, at 8D downstream, L=8D, Re=12.000

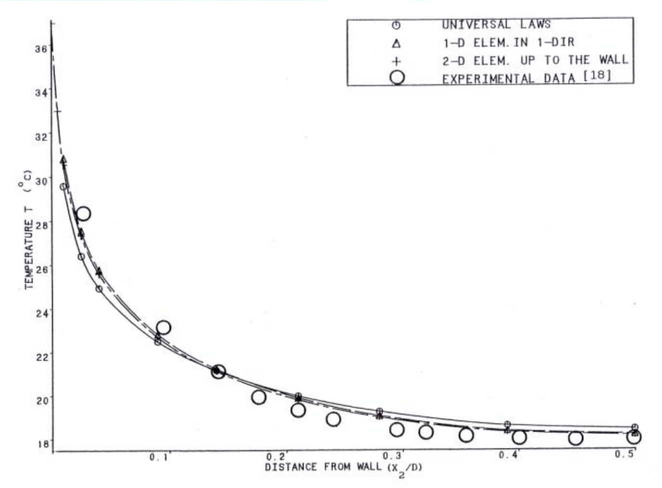

Figure 7: Temperature profiles for fully-developed flow, L=10D, at interface 0.49D, Re=12.000

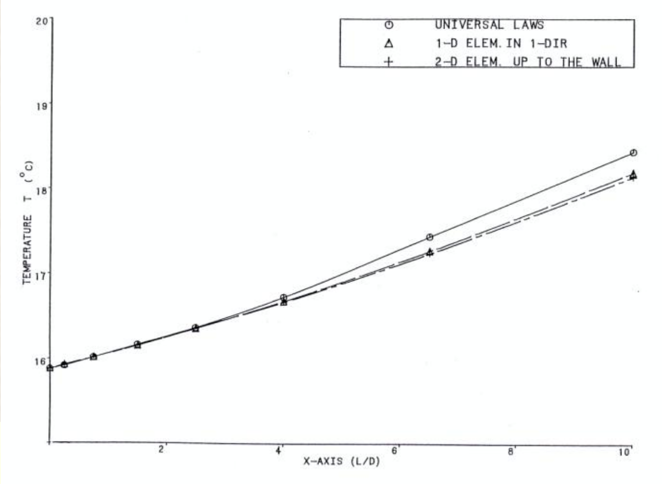

Figure 8: Temperature distribution along the centre line for fully-developed flow, L=10D, Re=12.000

V. Conclusions:

Since the use of 2-D elements up to the wall needs an excessive refinement which is not economically viable, and the utilization of universal laws are really applicable for certain unidimensional flow regimes which is not valid. These methods have been replaced by introducing a wall element technique, based on the use of the F.E.M. which has shown an excellent results when the turbulent fully-developed flow and heat transfer was considered and can be used with confidence.

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